CDs for the Band

Bryan and his band want to record and sell CDs. The recording studio charges an initial set-up fee of \$250, and each CD will cost \$5.50 to burn. The studio requires bands to make a minimum purchase of \$850, which includes the set-up fee and the cost of burning CDs.

1. Write a function rule relating the total cost and the number of CDs burned.

2. What are a reasonable domain and range for this problem situation?

3. Write and solve an inequality to determine the minimum number of CDs the band needs to burn to meet the minimum purchase of \$850.

4. If the initial set-up fee of \$250 is reduced by 50% but the cost per CD and the minimum purchase requirement do not change, will the new total cost be less than, equal to, or more than 50% of the original total cost? Justify your answer.

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Scaffolding Questions

- What is the total cost if the band purchases only one CD? Two CDs? Ten CDs?
- What are the variables? What do they represent?
- What are the constants for this situation?
- Describe in words the dependency relationship between the variables.
- What are the domain and range for this situation?
- What does the $850 represent in this situation?

Sample Solutions

1. Write a function rule relating the total cost and the number of CDs burned.

   The total cost of recording CDs is a $250 set-up fee plus the product of $5.50 and the number of CDs the band wants to purchase, so $C = 250 + 5.50n$, where C represents the total cost and n represents the number of CDs.

2. What are a reasonable domain and range for this problem situation?

   A reasonable domain for the situation is {1, 2, . . .}, since the number of CDs must be a whole number. The range values are the costs per CD at $5.50 each: {5.50, 11.00, 16.50, . . .}

3. Write and solve an inequality to determine the minimum number of CDs the band needs to burn to meet the minimum purchase of $850.

   The total cost must be greater than or equal to $850.

   Use the rule from the answer to question 1, and find the cost of various numbers of CDs.

Number of CDs	Total Cost
1	$255.50
10	$305.00
100	$800.00
110	$855.00

You know from the table that 110 CDs cost $855.00, which is just a little over the minimum fee of $850. Next, calculate the cost of 109 CDs using the rule; the total cost is $250 + $5.50(109) = $849.50.

At $849.50, 109 CDs cost less than $850, so the band must burn at least 110 CDs.

Using an inequality to solve the problem:

$$\begin{aligned} 250 + 5.50n &\geq 850 \\ 5.50n &\geq 850 - 250 \\ 5.50n &\geq 600 \\ n &\geq 109.09 \end{aligned}$$

CDs must be purchased in whole number quantities. Therefore, the band can burn 109 CDs for $849.50 but would have to pay another $0.50 to meet the minimum purchase requirement, or they could get 110 CDs for $855.

4. If the initial set-up fee of $250 is reduced by 50% but the cost per CD and the minimum purchase requirement do not change, will the new total cost be less than, equal to, or more than 50% of the original total cost? Justify your answer.

   If the set-up fee is reduced by 50%, it will be 0.50(250) or $125. The cost function becomes $C = 125 + 5.50n$.

   50% of the original cost can be calculated like this:

   $$0.50(250 + 5.50n) = 0.50(250) + 0.50(5.50)n = 125 + 2.75n$$

   $$125 + 2.75n \leq 125 + 5.50n$$

   The new cost is more than 50% of the original cost.

Extension Questions

- Suppose Bryan found another company that charges a set-up fee of $200 and $6.00 per CD. Would it be more economical for the band to purchase CDs from this company if it expects to spend at least $850?

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-IF) **Interpret functions that arise in applications in terms of the context**

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS Additional Teacher Content

(6.EE) **Represent and analyze quantitative relationships between dependent and independent variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

(8.F) **Define, evaluate, and compare functions.**

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

(F-IF) **Analyze functions using different representations**

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

4. Model with mathematics.

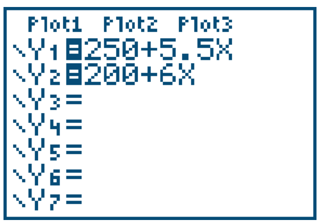
The cost function under these conditions is C = 200 + 6n.

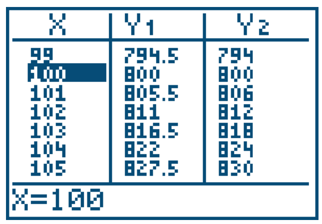
$$\begin{aligned} 200 + 6n &= 850 \\ 6n &= 850 - 200 \\ 6n &= 650 \\ n &= 108.33 \end{aligned}$$

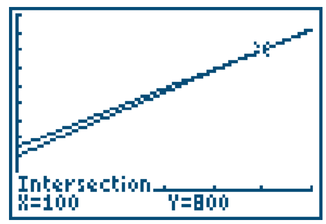
The band could purchase 108 CDs. This is not a better company to purchase from if the band plans to spend \$850 since it could get 109 CDs with the first company.

- Under what circumstances would the second company be a better choice for the band to use for producing their CDs?

The tables and graphs of the two functions may be compared to determine when they are equal in cost.







The functions have the same value when x is 100. The first company's cost is greater for values of x less than 100. The second company's cost is greater for values of x more than 100. However, the first company's restriction of spending a minimum of \$850 means that the first company is actually not a better deal unless the band wants to purchase more than 108 CDs.

Number of CDs	First Company Cost Y = 250 + 5.5x	First Company Cost with \$850 Minimum	Second Company Cost Y = 200 + 6x
100	800.00	850.00	800.00
105	827.50	850.00	830.00
106	833.00	850.00	836.00
107	838.50	850.00	842.00
108	844.00	850.00	848.00
109	849.50	850.00	854.00
110	855.00	855.00	860.00
111	860.50	860.50	866.00

The Shuttle's Glide

When space shuttles return to Earth for landing, they travel in a long glide. During a return flight, an observer records the shuttle's height above Earth during the glide, beginning when the shuttle is 100 km above Earth's surface.

time (minutes)	0	10	15	20	22
height (km)	100	97.2	95.8	94.4	93.84

1. Using symbols and words, describe the relationship between the time in minutes and the shuttle's height above Earth's surface in kilometers.
2. Assuming that the shuttle travels at a constant speed, how far above Earth was the shuttle 2 minutes before the observer began timing? Explain your answer.
3. How long before the observer began recording data was the shuttle at 102 km above Earth's surface? How do you know?

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine

Scaffolding Questions

- What are the variables in this situation?
- What would you expect the height above Earth's surface to be after 5 minutes? Explain your reasoning.
- Is the relationship linear? How can you tell?
- What is the rate of change?
- What is the y-intercept?

Sample Solutions

1. Using symbols and words, describe the relationship between the time in minutes and the shuttle's height above Earth's surface in kilometers.

   Determine the rates of change from the table.

   10 5 5 2

time (minutes)	0	10	15	20	22
height (km)	100	97.2	95.8	94.4	93.84

   -2.8 -1.4 -1.4 -0.56

   The rate of change is –0.28 km per minute.

   The height, h, is the starting height plus the rate of change, –0.28, times the number of minutes, m. The height of the shuttle in kilometers is 100 kilometers minus the product of 0.28 kilometers per hour and the number of minutes.

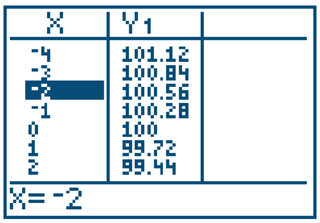
   $h = 100 - 0.28m$

2. Assuming that the shuttle travels at a constant speed, how far above Earth was the shuttle 2 minutes before the observer began timing? Explain your answer.

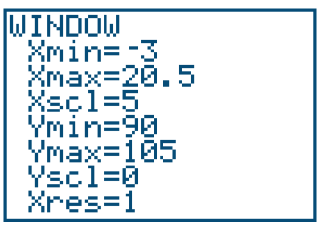
   If the descent had begun at this constant rate at least 2 minutes before the timing began, the time would be represented by $m = -2$. So the shuttle was at 100 – 0.28(–2) or 100.56 kilometers above Earth's surface.

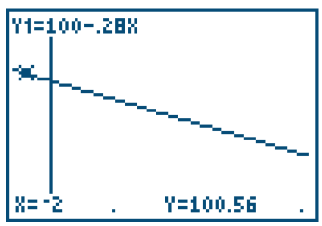
We could also use the table or the graph to find a solution.

If using a graphing calculator, students can translate the equation to $y = 100 - 0.28x$, where x represents the number of minutes and y represents the height in kilometers. Enter the rule $y = 100 - 0.28x$ into the graphing calculator and look at the table of values to find the value of y when $x = -2$.



The window can also be adjusted so that the graph can be traced to find the value when $x = -2$.





3. How long before the observer began recording data was the shuttle at 102 km above Earth's surface? How do you know?

   Students may approach this problem with a variety of strategies. For example, students may use the table in a graphing calculator and the equation $y = 100 - 0.28x$ to determine when y is 102. Students will find that they need to adjust the table settings to smaller and smaller increments for the x-values. Shown below are possible tables that students may generate—one using increments of 1 minute, another with increments of 0.1 minutes, and the third with increments of 0.01 minutes. The calculator will round the y-values to the nearest hundredth, so the closest estimate that can be obtained, in answer to the question of when the

the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(A-REI) **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS Additional Teacher Content

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

1.Distinguish between situations that can be modeled with linear functions and with exponential functions.*

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

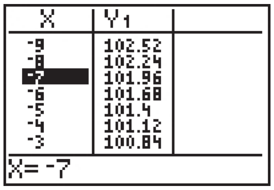
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

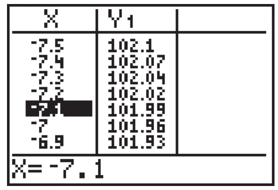
Standards for Mathematical Practice

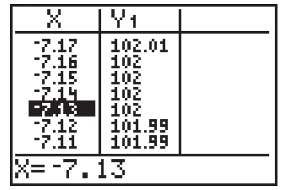
1. Make sense of problems and persevere in solving them.

4. Model with mathematics.

shuttle's altitude would be 102 kilometers, is that it would occur between –7.12 and –7.17 seconds.







Another strategy is to solve using the equation $102 = 100 - 0.28x$, finding that $x \approx -7.14$. This means that, if the shuttle were traveling at this constant rate, then 7.14 seconds prior to when the observer began recording data, the shuttle would have been 102 km above Earth's surface.

Extension Questions

- What does the *y*-intercept represent in this situation?

  The y-intercept represents the height at time zero. In other words, the y-intercept represents the height of the shuttle when the observer first began gathering data.

- What is the *x*-intercept, and what does it mean for this problem situation?

  The x-intercept is approximately 367.14. It represents the number of minutes after the observer began recording time that the height is 0—that is, the time when the shuttle would land if it continued to descend at the same rate.

- Is this a realistic model for the descent of a shuttle?

  This is not a realistic situation. In reality, the shuttle must decrease its speed as it gets closer to landing.

Making Pizzas, Making Money

The CTW Pizza Company plans to produce small, square pizzas. It will cost the company $2.00 to make each pizza, and they will sell the pizzas for $5.00 each.

1. Express the profit earned as a function of the number of pizzas sold.

2. Write a verbal description of the relationship between the two variables, and then represent the relationship with a table and a graph.

3. What is the slope of the graph? What does it mean in the context of the situation?

4. Describe at least two methods for finding the number of pizzas that need to be sold to make a profit of at least $180.

5. The CTW Pizza Company found a cheaper supplier, and now it costs $0.50 less to make each pizza. Describe how this changes the function rule, graph, and table, and explain how you know.

Notes

CCSS Content Task

(8.EE) **Understand the connections between proportional relationships, lines, and linear equations.**

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

(8.F) **Define, evaluate, and compare functions.**

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

(A-CED) **Create equations that describe numbers or relationships**

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example,*

Scaffolding Questions

- What is the profit for one pizza? For two pizzas? Three pizzas?
- What do the variables represent in this situation?
- Can you set up a table to help you determine the relationship between the variables?
- How much profit will be made by selling 50 pizzas?
- If the CTW Pizza Company's goal is to make a profit of at least $300 a day, how many pizzas must it sell each day?
- Is the relationship between profit and number of pizzas sold proportional? How do you know?

Sample Solutions

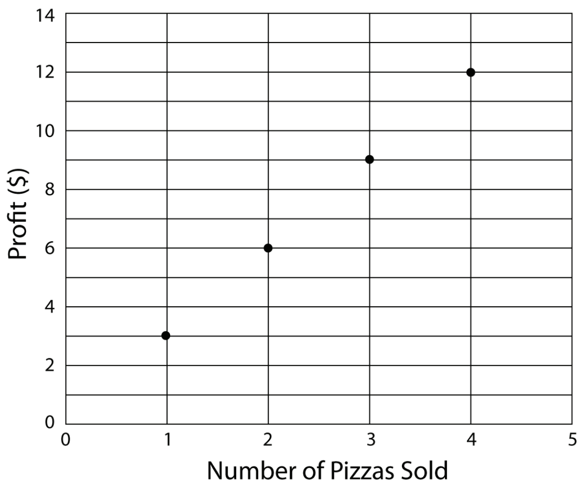
1. Express the profit earned as a function of the number of pizzas sold.

   The profit made from selling pizzas can be determined by subtracting the cost to make each pizza from the selling price. Therefore, the function rule for the profit, p, is $p = 5x - 2x$ or $p = 3x$, where x represents the number of pizzas.

2. Write a verbal description of the relationship between the two variables, and then represent the relationship with a table and a graph.

   The profit in dollars is the number of pizzas sold multiplied by $3.00. There is $3.00 profit per pizza. The more pizzas sold, the more profit made.

Number of Pizzas Sold	Profit ($)
1	3
2	6
3	9
4	12

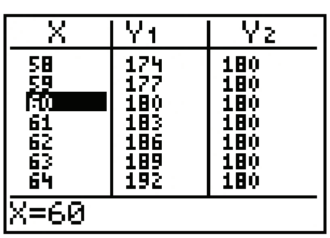


3. What is the slope of the graph? What does it mean in the context of the situation?

   The slope is the profit made in dollars per number of pizzas sold. The slope can be determined from the graph by looking for the rate of change: The profit goes up $3.00 for every pizza sold.

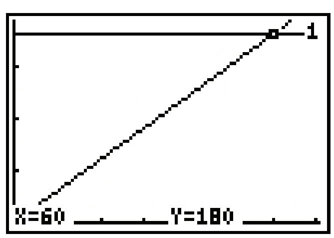
4. Describe at least two methods for finding the number of pizzas that need to be sold to make a profit of at least $180.

   Use the function $p = 3x$. We want to know when $3x$ is greater than $180, or in symbols: $180 \leq 3x$. Since 3 times 60 is 180, the company must sell at least 60 pizzas to make a profit of $180 or more, or in symbols $60 \leq x$. A table or graph can also be used to find y-values greater than or equal to 180. For the graph, let $Y2 = 180$, and then read from the graph where $Y1 \geq Y2$.



X	Y1	Y2
58	174	180
59	177	180
60	180	180
61	183	180
62	186	180
63	189	180
64	192	180

X=60



5. The CTW Pizza Company found a cheaper supplier, and now it costs $0.50 less to make each pizza. Describe how this changes the function rule, graph, and table, and explain how you know.

represent inequalities describing nutritional and cost constraints on combinations of different foods.

(F-IF) **Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Notes

CCSS Additional Teacher Content

(7.RP) **Analyze proportional relationships and use them to solve real-world and mathematical problems.**

2. Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

The new cost of making one pizza is \$1.50. The profit for each pizza is $5x - 1.50x$, or $3.50x$. The profit has increased by \$0.50 per pizza; therefore, the profit per pizza is now \$3.50. The y-intercept in the function rule $y = mx + b$ is still 0, but the slope increases by \$0.50. The slope of the graph is greater because for every pizza CTW sells it now makes \$3.50 instead of \$3.00. So the new equation for profit is $y = 3.50x$. The table also shows an increase of \$3.50 in the y-value for every increase in 1 of the x-value.

Extension Questions

- Describe how to determine the slope from your table in the answer to question 2.

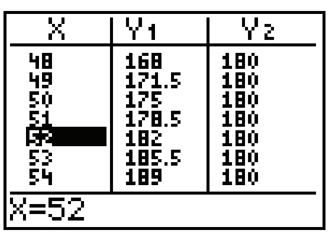
  Calculate the rates of changes by finding the difference of two y-values, divide by the difference between the corresponding x-values, and look for a constant rate of change.

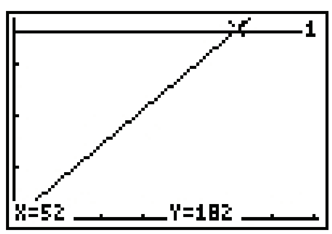
- Describe how to determine the slope from the graph.

  Determine the slope from the graph by finding the ratio of the vertical change to the horizontal change between any two points on the line.

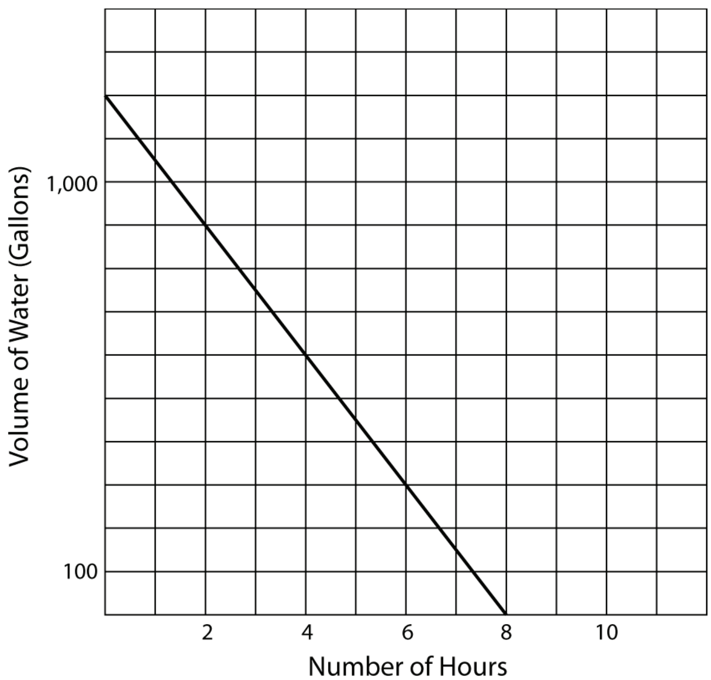
- If the CTW Pizza Company uses the cheaper supplier, how many fewer pizzas do they need to sell in order to make a profit of at least \$180?

  Using a graphing calculator with Y1 = 3.50x and Y2 = 180, we can learn that CTW needs to sell at least 52 pizzas to make a profit of at least \$180, which is 8 fewer pizzas than it needed with the first supplier.





Draining Pools



The graph shows the relationship between the amount of water in a pool and the number of hours that have elapsed since a pump began to drain the pool.

1. Describe verbally and symbolically the relationship between the amount of water in the pool and the number of hours that have elapsed since the draining began.

2. How much water is in the pool after 4 hours and 20 minutes? Describe your solution strategy.

3. How many hours after the pump began draining the pool does the pool contain 720 gallons of water? Describe two methods for determining your answer.

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

(8.EE) **Understand the connections between proportional relationships, lines, and linear equations.**

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

Scaffolding Questions

- Define the independent and dependent variables for this problem situation.
- What type of relationship does the graph represent?
- How much water was in the pool when the pumping started? What role will this number play in the function rule? What role will this number play in the graph?
- How much water remains in the pool after 2 hours? After 4 hours? After 6 hours? Organize your responses in a table.
- At what rate is the amount of water decreasing per hour?
- Use the rate of change and the starting volume in the pool to write a function rule.
- What are the domain and range for the problem situation?

Sample Solutions

1. Describe verbally and symbolically the relationship between the amount of water in the pool and the number of hours that have elapsed since the draining began.

   The amount of water in the pool at time 0 is 1,200 gallons.

   The graph is a straight line, which means the water is draining at a constant rate. It takes 8 hours to drain the pool completely—that is, to the point that 0 gallons remain. The rate of change per hour is 1,200 gallons divided by 8 hours, or 150 gallons per hour. Because the amount of water is decreasing, the rate of change is –150 gallons per hour.

   The amount of water in the pool is the starting value plus the product of the rate of change and the number of hours. (Or, for a draining pool, it is the starting value decreased by the product of the rate of 150 gallons per hour and the number of hours the pool has been draining.)

Let w be the amount of water in the pool at time t in hours.

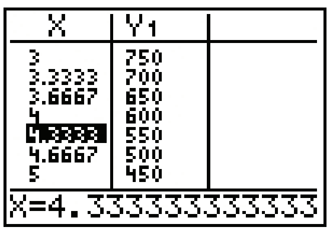
$w = 1{,}200 + (-150)t$

$w = 1{,}200 - 150t$, where t is any number from 0 to 8, inclusive

2. How much water is in the pool after 4 hours and 20 minutes? Describe your solution strategy.

The time is 4 hours and 20 minutes, or $4\frac{1}{3}$ hours.

We can obtain this answer using a graphing calculator if we set ΔTbl = 1/3 for the function $y = 1{,}200 - 150x$.



Or we can evaluate the function rule at $t = 4\frac{1}{3}$:

$w = 1{,}200 - 150(4\frac{1}{3}) = 1{,}200 - 650 = 550$

The amount of water in the pool after 4 hours and 20 minutes of draining is 550 gallons.

3. How many hours after the pump began draining the pool does the pool contain 720 gallons of water? Describe two methods for determining your answer.

When the amount of water left in the pool is 720 gallons, $y = 720$.

One way to solve the problem is by creating and solving an equation:

$$\begin{aligned} 720 &= 1{,}200 - 150t \\ t &= 3.2 \end{aligned}$$

There are 720 gallons of water in the pool after 3.2 hours, or 3 hours and 12 minutes.

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between

quantities; graph equations on coordinate axes with labels and scales.

(A-REI) **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-IF) **Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standards for Mathematical Practice

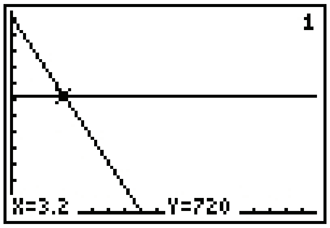
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

A table or graph can also be used to determine at what point the amount of water is 720 gallons. The equation 720 = 1,200 – 150x asks for where the graphs of two functions intersect:

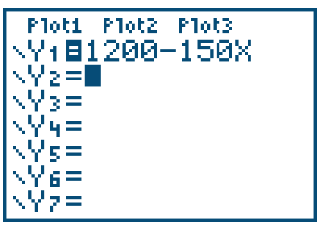
Y1 = 1,200 – 150x

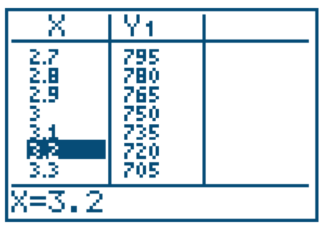
Y2 = 720

Graph and trace to find the solution.



To solve with a table, set the table minimum at 1 and increments at 0.1, and scroll down the table to find the value when y = 720 at x = 3.2. In 3.2 hours, the amount of water in the pool is 720 gallons.





Extension Questions

- What are the mathematical domain and range for the function rule you have written?

  For the general function rule not restricted by the draining pools scenario, the domain is the set of all real numbers. The range is the set of all real numbers.

- Describe the domain and range for this problem situation and explain why you selected this domain.

  The domain is the set of all real numbers from 0 to 8 inclusive, the values for the time elapsed. The domain values must be non-negative numbers and must give non-negative range values. The pool is empty after 8 hours.

  The range is the set of real numbers that show the amount of water (in gallons) in the pool: 0 to 1,200.

- How much time has elapsed if the pool is half empty?

  The original amount of water in the pool was 1,200 gallons, so when it is half empty, it contains 600 gallons. The pool ishalf empty at 4 hours. Note that this is one-half the time it takes to empty the pool.

- If half of the water drains in half the total time needed to empty the pool, predict how much of the total time has elapsed if one-third of the water has drained from the pool. Explain your reasoning.

  Draining one-third of the water takes one-third of the time it takes to drain the pool completely. There is a proportional relationship between the time and the portion of the water that has been drained.

  The time it takes to drain the pool is $\frac{1{,}200 \text{ gallons}}{150 \text{ gallons per hour}}$ *, or 8 hours.*

  If one-third of the pool is drained, two-thirds of the pool volume remains.

  $\frac{2}{3}(1{,}200) = 1{,}200 - 150x$

  $150x = \frac{1}{3}(1{,}200)$

  $x = \frac{1}{3} \bullet \frac{1{,}200}{150}$

  $x = \frac{1}{3}(8)$

  Let f be the fractional portion of the pool drained. The part remaining is 1 – f.

  $(1 - f)1{,}200 = 1200 - 150x$

  $150x = 1{,}200 - (1 - f)1{,}200$

  $150x = f1{,}200$

  $x = f(\frac{1{,}200}{150})$ *or 8f*

  Thus, if the amount of water drained is (f)(1,200), the time it takes to drain that amount of water is (f)(8), or f times the amount of time it takes to drain the pool.

- If the initial volume of the pool were 1,500 gallons, but the pool drained at the same rate, how would this affect your graph?

  The only value in the function that would change would be the y-intercept.

  $y = 1{,}500 - 150x$

  The graph would be a straight line parallel to the original line but with a y-intercept of 1,500 and an x-intercept of 10.

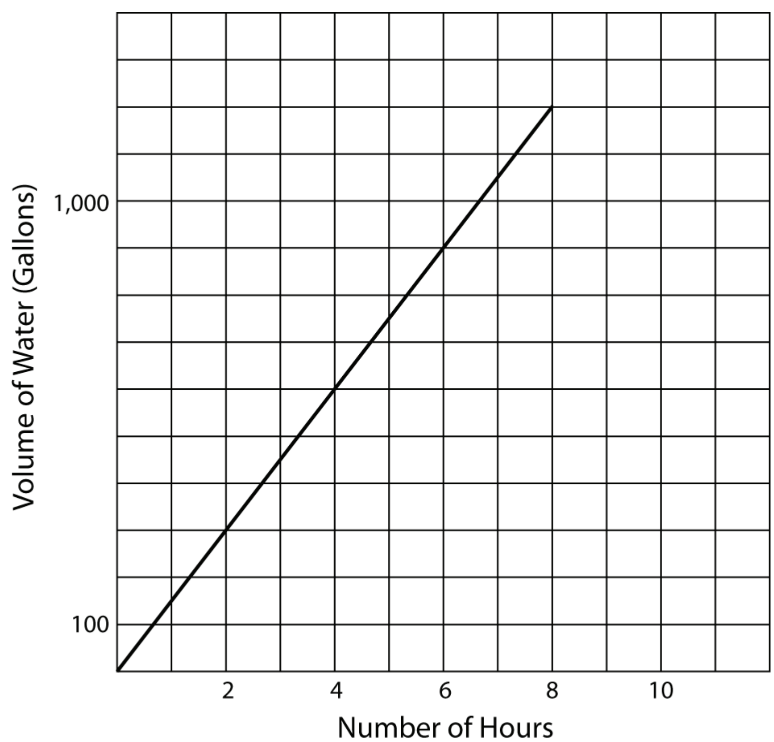
- If the pool started with the same amount of water (1,200 gallons) but emptied at 100 gallons per hour, how would the graph change?

The rate of change, or slope, would be –100. The function would be $y = 1{,}200 - 100x$.

Both the original graph and the new graph would have the same y-intercept, but since the new graph would descend at a slower rate, it would have a greater x-intercept than the original, meaning that it would take the pool longer to empty.

- Suppose an empty pool is filled at the same rate and with the same capacity of 1,200 gallons. Sketch the graph and write the function to represent this new situation.

  The function is $y = 150x$, where x varies from 0 to 8. Since the capacity of the pool is 1,200 gallons, the graph terminates at the point (8, 1,200); the graph is a line segment.



T-Shirts

Several school organizations want to sell t-shirts to raise money. Reynaldo, the treasurer for the Science Club, has found four different companies that can fill the t-shirt orders. Each function below represents the cost of placing a t-shirt order as a function of the number of t-shirts purchased.

Company A $c = 5t$

Company B $c = 3.25t + 55$

Company C $c = 3t + 100$

Company D $c = 6t - 55$

1. For each t-shirt company, write a verbal description or story that explains the company's method for calculating the cost of placing a t-shirt order. Be sure your description includes how each number in a particular function rule could be used to calculate the cost for that company.
2. Describe the differences in the methods used by Companies A and B. How are those differences represented in the function rules?
3. Make a table for each function, and then graph the four functions on the same graph.
4. Write two questions that could arise from the scenarios. Answer each one using either the graph or the table.
5. For each function, describe the difference in the domain for the function and the domain for your problem situation.
6. For each function, describe the difference in the range for the function and the range for the problem situation.

Notes

CCSS Content Task

(6.EE) **Reason about and solve one-variable equations and inequalities.**

7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

   b. Solve linear equations with rational number coefficients, including equations whose

Scaffolding Questions

- What are the variables in this scenario? Which is dependent? Independent?
- For Company A, what does the 5 represent?
- For Company B, which constant represents the cost per t-shirt?
- For Company B, what might the constant 55 represent?
- For Company C, what does the 3 represent?
- For Company C, what might the constant 100 represent?
- For Company D, which constant represents the cost per t-shirt?
- For Company D, what might the constant –55 represent?
- What are some things you might consider if you had to decide which company to use based on its pricing method?

Sample Solutions

1. For each t-shirt company, write a verbal description or story that explains the company's method for calculating the cost of placing a t-shirt order. Be sure your description includes how each number in a particular function rule could be used to calculate the cost for that company.

   Verbal descriptions or stories will vary. Examples:

   Company A. The Math Club treasurer made a deal with the manager of Company A. If the Math Club places an order, the cost will be $5.00 per shirt.

   Company B. The Spanish Club feels Company B offers a better deal because the club will get their t-shirts for only $3.25 each, although they do have to pay a $55.00 set-up fee.

   Company C. The Math Club found another deal, this time with Company C. They will pay only $3.00 per shirt with a $100.00 set-up fee.

Company D. The president of the freshman class thinks she has the best deal: Her father's friend, who runs Company D, will sell shirts for $6.00 each and give a $55.00 discount.

2. Describe the differences in the methods used by Companies A and B. How are those differences represented in the function rules?

   Companies A and B charge different amounts per shirt. This is represented as the coefficient of *x* in each function rule. Company A charges $5.00 per shirt, and Company B charges $3.25 per shirt. Company B also charges a fee of some kind as represented by "+55" in the function rule.

3. Make a table for each function, and then graph the four functions on the same graph.

Company A
$c = 5t$

t	c
0	0
10	50
20	100
30	150
40	200
50	250
60	300

Company B
$c = 3.25t + 55$

t	c
0	55
10	87.50
20	120
30	152.50
40	185
50	217.50
60	250

Company C
$c = 3t + 100$

t	c
0	100
10	130
20	160
30	190
40	220
50	250
60	280

Company D
$c = 6t - 55$

t	c
0	–55
10	5
20	65
30	125
40	185
50	245
60	305

solutions require expanding expressions using the distributive property and collecting like terms.

(8.F) **Define, evaluate, and compare functions.**

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-IF) **Interpret functions that arise in applications in terms of the context**

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-*

*hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

(F-IF) **Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

CCSS Additional Teacher Content

(6.EE) **Represent and analyze quantitative relationships between dependent and independent variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For

example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

(F-IF) **Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

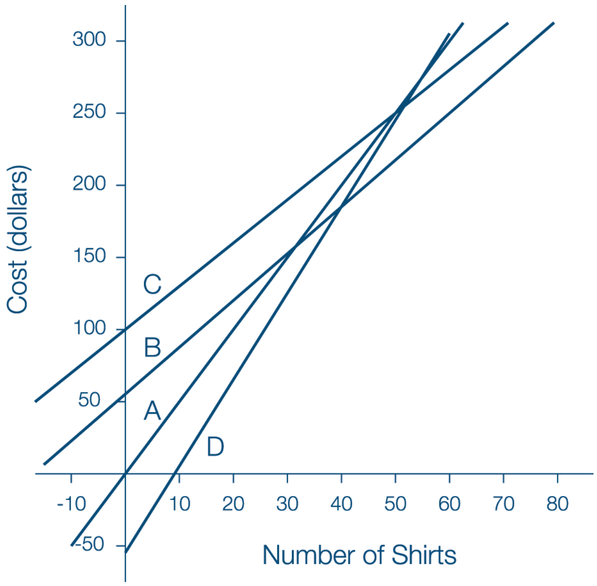
(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

6. Attend to precision.



4. Write two questions that could arise from the scenarios. Answer each one using either the graph or the table.

   Answers will vary. Examples of questions:

   - From which t-shirt company should a group purchase shirts if they plan to purchase 50 shirts?

     The table and graph show that the cost for 50 shirts is the least with Company B.

   - When does Company D give the better deal?

     The table and graph show that Company D is the better deal for up to 40 shirts.

   - Which company is never the least expensive?

     From the graph, we can see that Company A is never the least expensive. It also looks as if Company C is never the least expensive; however, since it charges the lowest rate per shirt, it will be the best deal if an organization wants to order at least 180 t-shirts.

5. For each function, describe the difference in the domain for the function and the domain for your problem situation.

   The domain of each function is all real numbers because each function is a linear function. For this problem situation, the domain values must be whole numbers because shirts cannot be purchased in fractions.

6. For each function, describe the difference in the range for the function and the range for the problem situation.

The range of each function is all real numbers. However, in this problem situation, the amounts are restricted to dollar values. For example, with Company B, the amounts must be $55 plus a whole number multiple of $3.25.

Extension Questions

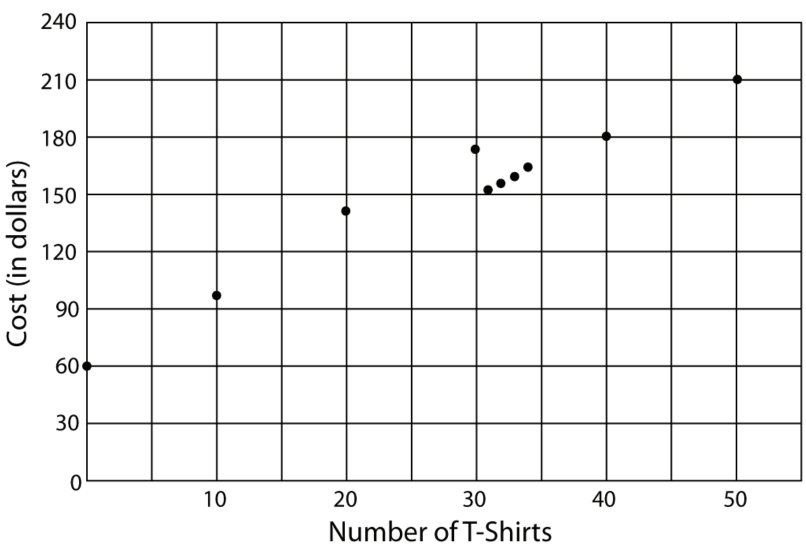
- If Company A had decided to give a discount of $40, how would that change the function rule?

  The function rule would become c = 5t – 40.

- Does Company B always offer the lowest cost?

  No. Company D charges the least for up to 40 shirts. Companies B and D have the same cost for 40 shirts. Company B charges the least for between 41 and 179 shirts. Companies B and C have the same cost at 180 shirts. For more than 180 shirts, Company C has the lowest cost. These cost breakdowns can be determined by examining the graph or table, or by solving symbolically.

- A new company, Company E, charges a set-up fee of $60 plus $3.75 for each t-shirt; however, if a customer orders more than 30 shirts, then Company E charges $60 plus $3.00 per t-shirt. Make a table and graph to represent Company E's pricing plan. Discuss how this company's plan differs from the others.



Number of T-shirts	Cost (in dollars)
0	*60*
10	*97.50*
20	*135*
30	*172.50*
31	*153*
32	*156*
33	*159*
34	*162*
40	*180*
50	*210*

This plan is different because the rate changes at 31 shirts.

Note to the teacher: *The Algebra I TEKS do not require students to write function rules for this type of scenario; however, for your information, the function is a combination of two linear functions:*

$$c = 60 + 3.75t \text{ for 0 to 30 shirts, and}$$

$$c = 60 + 3.00t \text{ for 31 or more shirts}$$

A function that consists of two functions is called a ***piecewise function****, and students formally learn about these in precalculus.*

Which Is Linear?

Four function rules were used to generate the following four tables:

I

x	y
–1	6
0	8
1	10
2	12
3	14

II

x	y
0	5
3	5
6	5
9	5
12	5

III

x	y
–2	–5
–1	–4.5
0	–4
3	–2.5
4	–2
5	–1.5

IV

x	y
–1	0.5
0	0
1	0.5
2	2
3	4.5
4	8
5	12.5

1. Which table or tables represent linear relationships? Explain how you decided.
2. Make a graph of the data in each table. Describe how the graphs are related.
3. Write a function rule for each linear relationship and explain how you developed each rule.

Notes

CCSS Content Task

(8.F) **Define, evaluate, and compare functions.**

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(A-CED) **Create equations that describe numbers or relationships**

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(F-IF) **Analyze functions using different representations**

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example,*

Scaffolding Questions

- As the x-values increase, what happens to the y-values?
- How are the patterns in the tables similar?
- How are the patterns in the tables different?
- What must be true about a function in order for it to be linear?
- How can you decide if a relationship is linear by looking at its table?
- What two numbers must you determine to write the linear function rule?

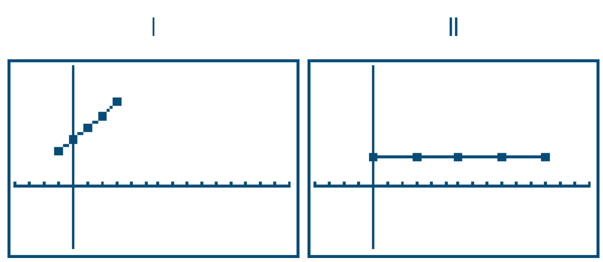
Sample Solutions

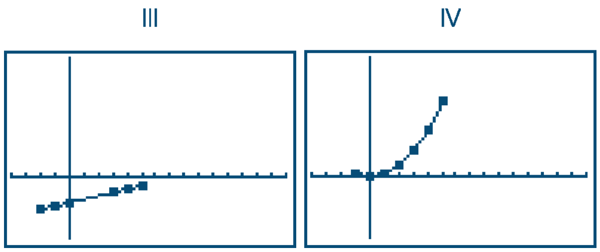
1. Which table or tables represent linear relationships? Explain how you decided.

   In Table I, as x increases by 1, y increases by 2. In Table II, as x increases by 3, y stays constant. In Table III, as x increases by 1, y increases by 0.5. In Table IV, there is not a constant rate of change. Therefore, Tables I, II, and III represent linear relationships. The graphs of these sets of points form lines. As x increases by a constant number, y also increases by a constant number.

2. Make a graph of the data in each table. Describe how the graphs are related.

   The scatterplots of the data are shown below in connected mode.





Three of the graphs show a linear relationship: I, II, and III. The graph of Table IV is not linear. The graph of Table I has the steepest line. The graph of Table II has a slope of zero.

3. Write a function rule for each linear relationship and explain how you developed each rule.

   In Table I, the rate of change is 2 because as x increases by 1, y increases by 2. The point (0, 8) indicates that the line crosses the y-axis at 8, so 8 is the y-intercept.

   The function rule for Table I is $y = 8 + 2x$, or $y = 2x + 8$.

   In Table II, the rate of change is 0 because there is no change in y as x changes. The point (0, 5) shows where the line crosses the y-axis. The function rule for Table II is $y = 5$.

   In Table III, the rate of change is 0.5 because the ratio of the change in y to the change in x is 0.5. The point (0, –4) indicates that the line intersects the y-axis at –4. The function for Table III is $y = 0.5x - 4$, or $y = -4 + 0.5x$.

Extension Questions

- For each function rule you wrote for your answers to question 3, describe a real-world scenario for which that function rule might be used. Also describe the domain and range for each scenario.

  Answers will vary. Students may determine separate scenarios for each function rule, or they may suggest a scenario that includes all the function rules. Sample responses:

given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS Additional Teacher Content

(F-IF) **Interpret functions that arise in applications in terms of the context**

5. Relate the domain of a

Notes

function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

5. Use appropriate tools strategically.

Last year, three sisters opened savings accounts at a nearby bank. Recently, each of them decided to check their balances weekly, and two of them will start adding money to their savings accounts each week.

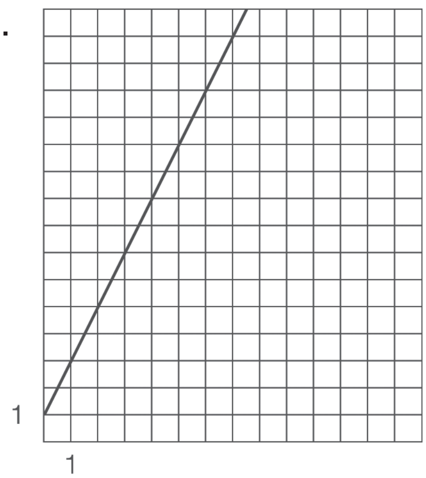
Rachel started with $8.00 in her account and will add $2.00 each week (Table I). Roxanne has $5.00 in her account and does not want to add any money each week (Table II). Rene was recently charged an annual fee, so she has –$4.00 in her account and plans to deposit $0.50 per week (Table III).

The domain for each sister's scenario is the number of weeks since they started checking their balance and is therefore whole numbers. The ranges for each sister will be the amount of money in each account. Rachel: {8.00, 10.00, 12.00, . . .}, Roxanne: {5.00}, Rene: {–4.00, –3.50, –3.00, . . .}

Finding Pairs

Six different functions are represented below. Compare and contrast the function rules, tables, graphs, and verbal descriptions. Identify which pairs represent the same functional relationship and explain how you know.

1. $y = 2x - 1$

2. 

3. The plant was growing at a rate of $1\frac{1}{2}$ inches per week.

4. The Math Club's treasurer found a place that will make the club custom t-shirts for $5.00 each, but there is a set-up fee of $50.

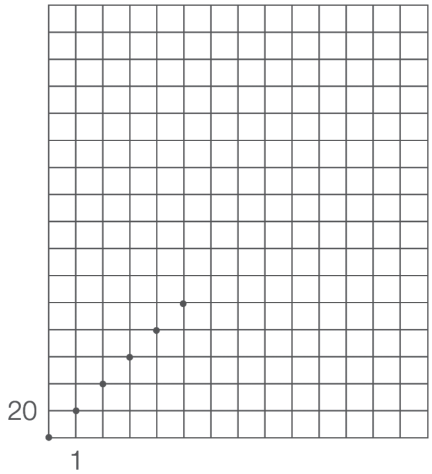
5.

x	*y*
0	50
1	55
2	60
3	65
4	70
5	75

6. $y = 2x + 1$

7. $f(x) = 1.5x$

8.



9.

x	*y*
–2	–5
–1	–3
0	–1
1	1
2	3
3	5

10.

x	*y*
–2	1.5
–1	1.5
0	1.5
1	1.5
2	1.5
3	1.5

11.

x	y
0	0
1	20
2	40
3	60
4	80
5	100

12. $y = 1.5$

Notes

CCSS Content Task

(8.F) **Define, evaluate, and compare functions.**

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

(8.F) **Use functions to model relationships between quantities.**

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*
   a. Interpret parts of an expression, such as terms, factors, and coefficients.

(F-IF) **Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch

Scaffolding Questions

- What type of relationship is described by each graph, table, function rule, and/or verbal description?
- What patterns do you notice in the tables? What is causing these patterns?
- How can you determine if there is a constant rate of change in a graph? A table?
- How is a constant rate of change represented in a verbal description? An equation or function rule?
- In an equation or function rule, what do the coefficients of x represent? If another number is added or subtracted, what does it represent?
- Can you make a connection between the graph in number 2 and any of the tables? Equations? Verbal descriptions?

Sample Solutions

The following pairs represent the same functional relationship:

Table 5 and Situation 4

The $50 set-up fee matches the entry $x = 0$, $y = 50$ in Table 5. The $5.00 per shirt matches the increments of 5 in the y-values for Table 5.

Table 9 and Function Rule 1

The point (0, –1) from Table 9 indicates a y-intercept of –1. The increase of 2 in the y-values, for every corresponding increase of 1 in the x-values, means that the slope is 2. The function rule is $y = 2x - 1$.

Table 11 and Graph 8

The point (0, 0) indicates that the graph of the function passes through the origin. In Table 11, the slope of the line is 20, since for every unit increase in x, y increases 20 units.

Table 10 and Function Rule 12

Every *y*-value in Table 10 is 1.5, which means *y* is always equal to 1.5.

Function Rule 6 and Graph 2

The graph of the function passes through point (0, 1), which means that the *y*-intercept is 1. The coefficient of *x* is 2, and 2 is also the slope of the line in Graph 2.

Function Rule 7 and Situation 3

The coefficient of *x* is 1.5 in Function Rule 7, and that is the same as the rate of change for the plant.

Extension Questions

- How do the numbers in the function rules affect the tables?

  The coefficient, m, of x in the function y = mx + b is the slope or rate of change that can be determined from the table. The constant, b, in the function corresponds to the y-intercept, the data point (0, b).

- Make a list of patterns you notice in the tables, and explain what causes the patterns.

  In Table 5, as x increases by 1, the values of y increase by 5. This is because the constant rate of change is 5. In a function rule, 5 will show up as the coefficient of x.

  The constant rate of change for Table 9 is 2.

  The constant rate of change for Table 10 is 0. The y-values do not change as x changes. In the equation y = mx + b, the slope is zero. Therefore, y = 0x + b or y = b for every x. The constant rate of change shows that the coefficient of x is 0. This indicates that the matching function does not have a term with an x.

  The constant rate of change for Table 11 is 20.

graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

(F-IF) **Analyze functions using different representations**

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

First Aid Supplies

Mark is the trainer for the Little Kids soccer team. He needs to buy 50 bandages and 3 ice packs for the team's first-aid kit. Mark can spend no more than $12. Prices vary for different ice packs, but every brand of bandage costs the same: $4.50 for 50 bandages. The sales tax is 9%.

1. Write an inequality to identify the amount Mark can spend on ice packs. Identify your variable.
2. What is the most Mark can spend for each ice pack and keep within the $12 budget? Show how you know.
3. Suppose the booster club gives Mark another $10 to spend on ice packs. How would this change the inequality that you wrote and your answer to question 2? Describe your solution verbally and algebraically.

Scaffolding Questions

- Is it possible for Mark to buy the bandages and ice packs for less than $12?
- How does the tax rate affect your inequality?

Sample Solutions

1. Write an inequality to identify the amount Mark can spend on ice packs. Identify your variable.

   The cost of the bandages is $4.50. The total cost of the supplies depends on the price of the ice pack. Let x = the price of 1 ice pack.

   Since Mark needs 3 ice packs, the cost of the ice packs is 3 multiplied by the price of 1 ice pack, or $3x$. The expression for the cost of the ice packs plus the cost of the bandages is $3x + 4.50$.

   The sales tax is 9% of the cost, or $0.09(3x + 4.50)$.

   The total cost including the tax must be less than or equal to $12:

   $(3x + 4.50) + 0.09(3x + 4.50) \leq 12.00$,
   or $1.09(3x + 4.50) \leq 12$

   Other students may identify the variable as the total amount that can be spent on ice, i. An inequality for that way of thinking about this scenario might be:

   $1.09(4.50 + i) \leq 12$

2. What is the most Mark can spend for each ice pack and keep within the $12 budget? Show how you know.

   Using the first equation above, students may solve this way:

$$\begin{aligned} 3x + 4.50 + 0.27x + 0.41 &\leq 12.00 \\ 3.27x + 4.91 &\leq 12.00 \\ 3.27x &\leq 7.09 \\ x &\leq 2.168195719 \end{aligned}$$

   x represents a dollar amount and must be expressed to the nearest hundredth. However, if you round up to

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

   b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*

   a. Interpret parts of an expression, such as terms, factors, and coefficients.

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

\$2.17, the cost of supplies is more than \$12:

$$3(2.17) + 4.50 + 0.09(3(2.17) + 4.50) = 12.009$$

Mark can spend no more than \$12, so he can pay up to \$2.16 per ice pack.

Using the second equation above, students would get that $i \leq 6.50$ if the amount is rounded to the nearest hundredth. Dividing the total amount that can be spent on ice, Mark would have $6.50 \div 3 \approx 2.17$. However, if each bag were \$2.17, that would be more than \$6.50, so Mark can only spend \$2.16 per pack of ice.

3. Suppose the booster club gives Mark another \$10 to spend on ice packs. How would this change the inequality that you wrote and your answer to question 2? Describe your solution verbally and algebraically.

If the booster club gave Mark an additional \$10 to spend for the ice packs, the only difference in the solution would be the total amount budgeted for purchase—\$22 rather than \$12. The additional money would allow Mark to purchase ice packs that are more expensive or to buy more bags of ice.

Let x = the price of an ice pack.

The total cost may now be less than or equal to \$12.00 plus \$10.00.

$$\begin{aligned}(3x + 4.50) + 0.09(3x + 4.50) &\leq 22.00\\ 3x + 4.50 + 0.27x + 0.41 &\leq 22.00\\ 3.27x + 4.91 &\leq 22.00\\ 3.27x &\leq 17.09\\ x &\leq 5.226\end{aligned}$$

If this answer is rounded to \$5.23 and Mark spent \$5.23 per ice pack, the cost would be \$27.01, so he can spend at most \$5.22 per ice pack. He may also end up buying more ice packs, because more expensive packs might mean heavier packs, and the team might need lighter packs or just a greater number of packs.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

6. Attend to precision.

Extension Questions

- Suppose Mark found the bandages on sale at 50 bandages for $3, and he could spend no more than $15. How much could he spend per ice pack?

  Let x = the price of an ice pack.

$$\begin{aligned}(3x + 3.00) + 0.09(3x + 3.00) &\leq 15.00\\ 3x + 3.00 + 0.27x + 0.27 &\leq 15.00\\ 3.27x + 3.27 &\leq 15.00\\ 3.27x &\leq 11.73\\ x &\leq 3.587\end{aligned}$$

  If the bandages are on sale for $3 and Mark can spend up to $15, he could pay up to $3.58 per ice pack.

- How would a 7% tax rate influence Mark's purchase if the bandages are still on sale for $3 and he has a maximum of $15?

  Let x = the price of an ice pack.

$$\begin{aligned}(3x + 3.00) + 0.07(3x + 3.00) &\leq 15.00\\ 3x + 3.00 + 0.21x + 0.21 &\leq 15.00\\ 3.21x + 3.21 &\leq 15.00\\ 3.21x &\leq 11.79\\ x &\leq 3.673\end{aligned}$$

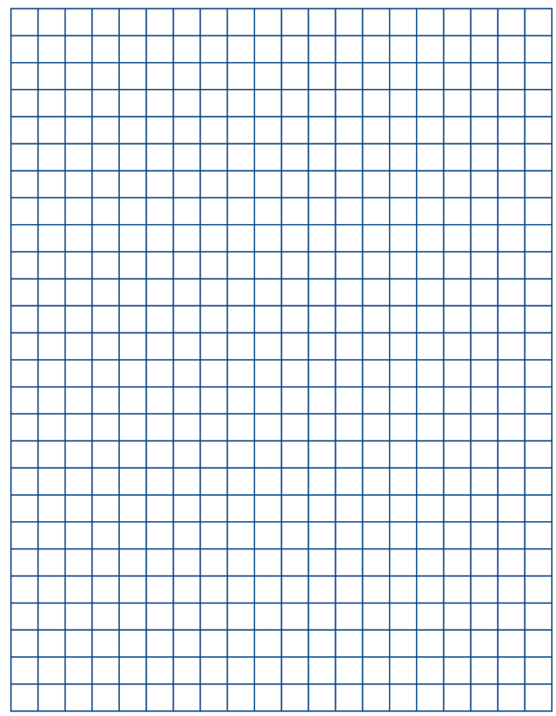
  Mark could spend up to $3.67 per ice pack, or $0.09 more per pack, if the tax rate were 7%.

Stretched Spring

The table below reports a sample of the data collected in an experiment to determine the relationship between the length of a spring and the mass of an object hanging from it. The length of the spring depends on the mass of the object.

Length versus Mass

Mass (kg)	Length (cm)
50	5.0
60	5.5
70	6.0
80	6.3
90	6.8
100	7.1
110	7.5
120	7.7
130	8.0
140	8.6
150	8.8
160	9.2
170	9.5
180	9.9
190	10.3



1. Construct a scatterplot of the data. Describe verbally and symbolically the functional relationship between the length of the spring and the mass suspended from it.
2. Predict the length of the spring when a mass of 250 kilograms is suspended from it. Describe the method you used to make your prediction.
3. Predict the mass that would stretch the spring to 15 centimeters. Explain your reasoning.

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine

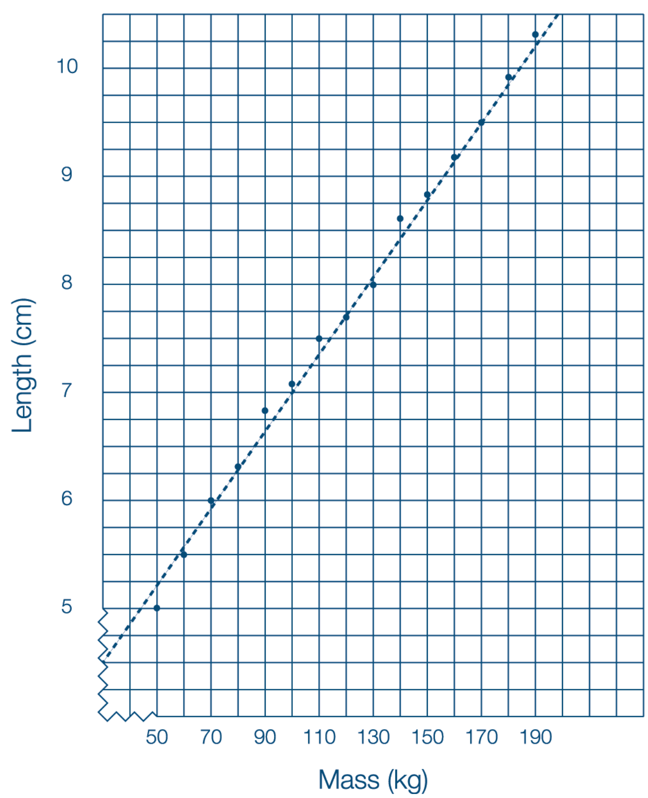
Scaffolding Questions

- How can you organize the data?
- What do you need to consider when constructing a scatterplot of the data?
- What do you need to consider when determining a reasonable interval of values and scale for each axis?
- Which function type (linear, quadratic, exponential, inverse variation) appears to best represent your scatterplot?
- What do you need to know when determining a particular function model for your scatterplot?

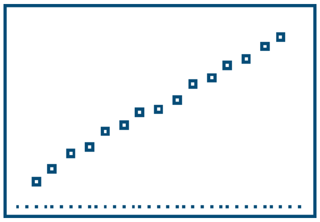
Sample Solutions

1. Construct a scatterplot of the data. Describe verbally and symbolically the functional relationship between the length of the spring and the mass suspended from it.

The scatterplot is nearly linear.



The data points may also be entered into a graphing calculator to create the scatterplot.



The consecutive difference in the length values may be computed using the list feature.

L1	L2	L3
50	5	------
60	5.5	
70	6	
80	6.3	
90	6.8	
100	7.1	
110	7.5	

L3 =ΔList(L2)

L1	L2	L3
50	5	.5
60	5.5	.5
70	6	.3
80	6.3	.5
90	6.8	.3
100	7.1	.4
110	7.5	.2

L3 ={.5,.5,.3,.5…

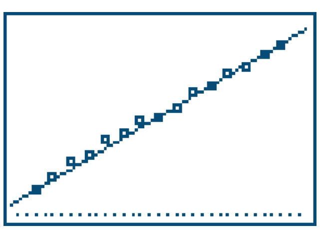
The average (mean) of these consecutive differences is approximately 0.38. The difference in the consecutive mass values is 10. The rate of change may be approximated as 0.38 divided by 10, or 0.038. Using 0.038 centimeters per kilogram as the rate of change, a trend line is of the form $y = 0.038x + b$. Use any other data point to find a possible value for b. If the point (50, 5) is used, the value is 3.1.

$5 = 0.038(50) + b$

$b = 5 - 0.038(50) = 3.1$

$y = 0.038x + 3.1$

The graph of this line is an approximate trend line for the data.



(Note: Students may discover that they can use the regression feature of the calculator to find a line of best fit. However, this calculator feature seems like a trick unless teachers help students understand the regression concept. Remember, this lies beyond the scope of Algebra I.)

the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(A-REI) **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS Additional Teacher Content

(8.F) **Define, evaluate, and compare functions.**

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = *s*2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

4. Model with mathematics.

2. Predict the length of the spring when a mass of 250 kilograms is suspended from it. Describe the method you used to make your prediction.

   To predict the length of the spring, evaluate the function for $x = 250$.

   $y = 0.038(250) + 3.1$

   $y = 12.6$ cm

   A mass of 250 kilograms will stretch the spring to an approximate length of 12.6 centimeters.

3. Predict the mass that would stretch the spring to 15 centimeters. Explain your reasoning.

   To predict the mass that would stretch the spring to 15 centimeters, use the function rule and solve the resulting equation:

$$\begin{aligned} 0.038x + 3.1 &= 15 \\ 0.038x &= 11.9 \\ x &= 313.16 \text{ kg} \end{aligned}$$

   A mass of about 313.16 kilograms stretches the spring to a length of 15 centimeters.

Extension Questions

- In the experiment, how did the mass suspended from the spring change and, in general, how did this affect the length to which the spring stretched?

  The mass suspended from the spring started at 50 kilograms and increased by 10 kilograms at intervals until it reached a mass of 190 kilograms. The initial amount of stretch (at 50 kilograms mass) was 5 centimeters, and the stretch increased by small amounts (0.2 to 0.5 centimeters) with each additional 10 kilograms of mass.

- With each 10-kilogram increase in mass, the spring stretched an additional 0.2 to 0.5 centimeters. What does this suggest about the functional relationship between spring length and mass?

  The relationship is approximately linear. As the mass increases in constant amounts, the additional length that the spring stretches is nearly constant. This suggests that a constant rate of change can be used to model the situation.

- How long is the spring when no mass is suspended from it?

  Use the function that models the situation, $y = 0.038x + 3.1$. When the spring has no mass attached to it, the value of x is 0, and y is 3.1 centimeters long.

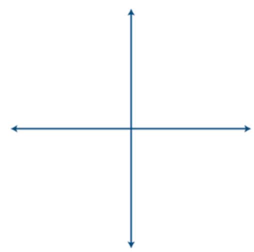
- Suppose the initial length of the spring is changed to 6.8 centimeters, and we suspend mass from the spring in increments of 20 kilograms instead of 10. How would this change the function that models this situation?

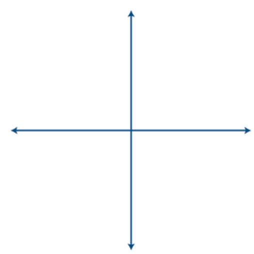
  Increasing the weight increments to 20 kilograms would not significantly affect how much the spring stretches; collecting data for different weights would not change the function that models the situation. If the spring has the same stretching ability—and the weights attached to it are within reasonable physical constraints—the rate of change would still be 0.038 centimeters per kilogram of mass. Changing the initial length of the spring to 6.8 centimeters would change the y-intercept to 6.8. The function that models the situation would be $y = 0.038x + 6.8$.

Analysis of a Function

I. If given a function rule, sketch a complete graph that represents that function. Show the coordinates of any intercepts. If given a graph or table, write the function representing it.

II. Determine the domain and range for each mathematical situation.

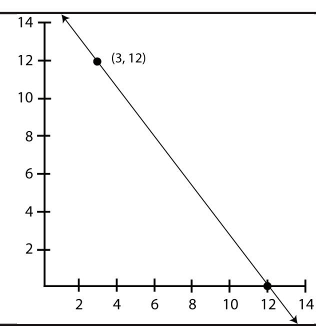
<table>
<tr><th></th><th>Function</th><th>Graph or Table</th><th>Domain and Range</th></tr>
<tr><td>1.</td><td>$f(x) = 5 - 2x$</td><td></td><td>Domain:

Range:</td></tr>
<tr><td>2.</td><td>$y = -2$</td><td></td><td>Domain:

Range:</td></tr>
<tr><td>3.</td><td></td><td>

x	y
−2	2
2	4
6	6
12	9

</td><td>Domain:

Range:</td></tr>
<tr><td>4.</td><td></td><td>
</td><td>Domain:

Range:</td></tr>
</table>

III. Describe the similarities and differences among the functions given above.

IV. Describe a practical situation that each function might represent. What restrictions on the mathematical domain and range of the function does the situation require? How does the situation affect the graph of the mathematical function?

Notes

CCSS Content Task

(8.EE) **Understand the connections between proportional relationships, lines, and linear equations.**

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

(8.F) **Define, evaluate, and compare functions.**

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of

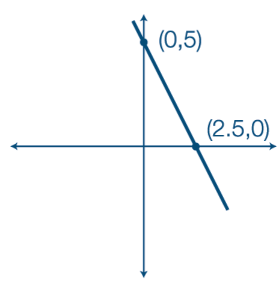
Scaffolding Questions

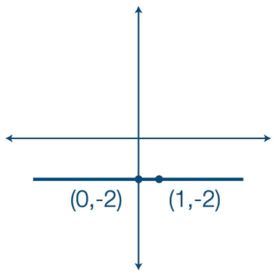
- What type of function relates the variables?
- What is the dependent variable? What is the independent variable? How do you know?
- What are the constants in the function? What do they mean?
- For the table in number 3, what restrictions does the function place on the independent variable?
- What is a reasonable domain for the function?
- What is a reasonable range for the function?

Sample Solutions

I. If given a function rule, sketch a complete graph that represents that function. Show the coordinates of any intercepts. If given a graph or table, write the function representing it.

II. Determine the domain and range for each mathematical situation.

	Function	Graph or Table	Domain and Range
1.	$f(x) = 5 - 2x$		Domain: The domain is the set of all real numbers because $5 - 2x$ is defined for any value of x. Range: The range is the set of all real numbers since any number can be generated by $5 - 2x$.

2.	$y = -2$		Domain: The domain is the set of all real numbers because the function y = –2 means "*y* is equal to –2 no matter what *x* is." This is a constant function. Range: *y* is always –2, thus the range is only the number –2.
3.	The table shows a constant rate of change of $\frac{1}{2}$, so these data model a linear function. The *y*-intercept is (0, 3). The function that models this set of points is $y = \frac{1}{2}x + 3$.	x | y –2 | 2 2 | 4 6 | 6 12 | 9	Domain: The domain is the set of given *x* values {–2, 2, 6, 12}. The range is the set of *y* values {2, 4, 6, 9}. The domain and range of the function that models these data are both the set of all real numbers.

its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

(A-CED) **Create equations that describe numbers or relationships**

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(F-IF) **Interpret functions that arise in applications in terms of the context**

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

(F-IF) **Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic*

expression for another, say which has the larger maximum.

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

CCSS Additional Teacher Content

(6.EE) **Represent and analyze quantitative relationships between dependent and independent variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.

Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

(8.F) **Define, evaluate, and compare functions.**

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. [3]

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = *s*2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

(F-IF) **Understand the concept of a function and use function notation**

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

[3] Function notation is not required in Grade 8.

Notes

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

1.Distinguish between situations that can be modeled with linear functions and with exponential functions.*

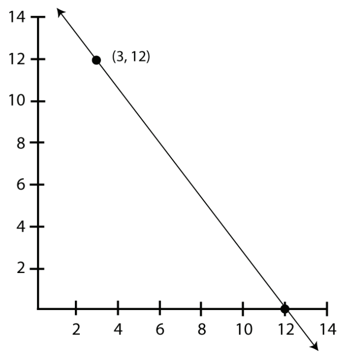
a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

4.	$y = -\frac{4}{3}x + 16$		Domain: The domain is the set of all real numbers because $y = -\frac{4}{3}x + 16$ is defined for any value of x. Range: The range is the set of all real numbers since any number can be generated by $y = -\frac{4}{3}x + 16$.

III. Describe the similarities and differences among the functions given above.

The functions in problems 1–4 are linear, having the form $y = mx + b$. They all have as their domains the set of all real numbers because the expression for each function is never undefined. The functions in problems 1, 3, and 4 have as their ranges the set of all real numbers, since every real number can be generated by the expressions for those functions. The function in problem 2 has as its range the single number –2, since it is a constant function.

The graphs of the functions in problems 1–4 are lines. The graph of the function in problem 1 has a y-intercept of (0, 5) and an x-intercept of (2.5, 0). The line falls from left to right because the slope is negative. This is a decreasing function. The graph of the function in problem 2 has a y-intercept of (0, –2) and no x-intercept. It is a horizontal line with slope zero. This is a constant function.

The graph of the function that models the data given in problem 3 has a y-intercept of (0, 3) and an x-intercept of (–6, 0). The line rises from left to right because the slope is positive. This is an increasing function.

The graph of the function in problem 4 has a y-intercept of (0, 16) and an x-intercept of (12, 0). The line decreases from left to right because the slope is negative. This is a decreasing function.

IV. Describe a practical situation that each function might represent. What restrictions on the mathematical domain and range of the function does the situation require? How does the situation affect the graph of the mathematical function?

The function $f(x) = 5 - 2x$ could represent a toy racecar starting to race 5 feet away from the finish line and moving forward at 2 feet per second, where y is the distance in feet between the toy car and the finish line and x is the time in seconds the car has been moving. For this situation, the domain is the set of all numbers x, $0 \le x \le 2.5$, representing the time to start and complete the race. The range is the set of all numbers y, $0 \le y \le 5$, representing the range of distance traveled by the car. The graph is simply the segment from (0, 5) to (2.5, 0).

The function $y = -2$ could represent an ocean diver in the waters near a beach. The diver is floating 2 meters below the water surface. For this situation, the domain is time, x in minutes, that the diver is at this depth. For example, the domain could be the set of all numbers x, $0 \le x \le 15$, and the range is $y = -2$. The graph is a horizontal segment from (0, –2) to (15, –2).

The function $y = \frac{1}{2}x + 3$ could represent the allowance that a very young boy gets each week. The parent puts \$3 in the boy's piggy bank to start. Each week, the boy gets a 50-cent allowance and adds it to the piggy bank. The boy has been told that if he saves his allowance each week for 6 months, then he will get an increase. For this situation, the domain is the set of all values x, $x = 0, 1, 2, 3, \ldots, 24$, because there are roughly 24 weeks in the 6-month period. The range is the set of all values y, $y = 3, 3.5, 4, 4.5, \ldots, 15$, representing the amount of the boy's savings.

The graph is a discrete graph because it is simply a plot of a set of 25 points.

The fourth function could represent the remaining life of the batteries in an MP3 player. The batteries are designed to last up to 16 hours, and Jessica listens to her MP3 player 80 minutes per day as she walks to and from school. She can use the function rule to determine the number of days that the battery will last if she uses the player for 80 minutes or $\frac{4}{3}$ of an hour each day.

Extension Questions

- For problems 1–3, what is the equation of a line perpendicular to each of the given lines and having the same y-intercept?

  If a function is not a horizontal line, find the slope of the line and determine the opposite reciprocal of this slope. If the line is a horizontal line, the perpendicular line has undefined slope.

  In problem 1, the line's slope is –2; the slope of a line perpendicular to this line is $\frac{1}{2}$. The equation of the line is $y = 5 + \frac{1}{2}x$.

In problem 2, the line is horizontal with y-intercept –2. The line perpendicular to this line is a vertical line. The slope of a vertical line is undefined. The line is of the form x equals a constant. Any vertical line is perpendicular to y = –2. The equation is x = k, where k is any real number.

For problem 3, the slope of the line is $\frac{1}{2}$. The perpendicular line has slope of –2. The equation of the line is y = –2x + 3.

- Describe the domain and range of these three perpendicular lines.

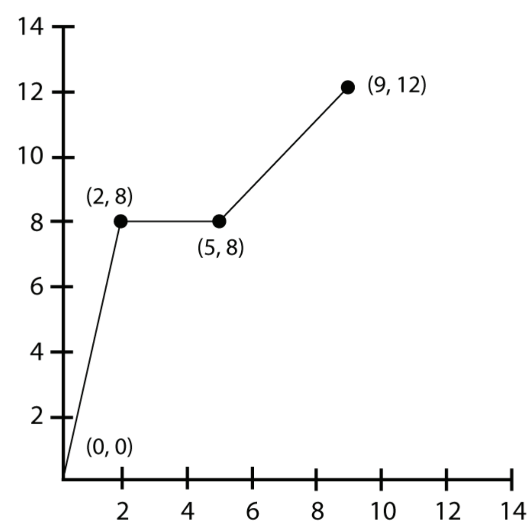
  The domain and range of the perpendicular lines in problems 1 and 3 are all real numbers.

  The domain of the line x = k is the number k. The range is all real numbers.

- Do these perpendicular lines represent functions?

  The perpendicular lines in problems 1 and 3 represent functions because for each x, there is only one y value. However, x = k does not represent a function because the x value 0 is paired with an infinite number of y values.

- Write a function rule for each phase of the graph below. Then determine the domain and range.



This function consists of three linear pieces. These could be defined as:

If $0 \le x \le 2$, $y = 4x$.

If $2 < x \le 5$, $y = 8$.

If $5 < x \le 9$, $y = x + 3$.

Domain: The domain is the set of all real numbers x, $0 \le x \le 9$, since this is what the graph shows.

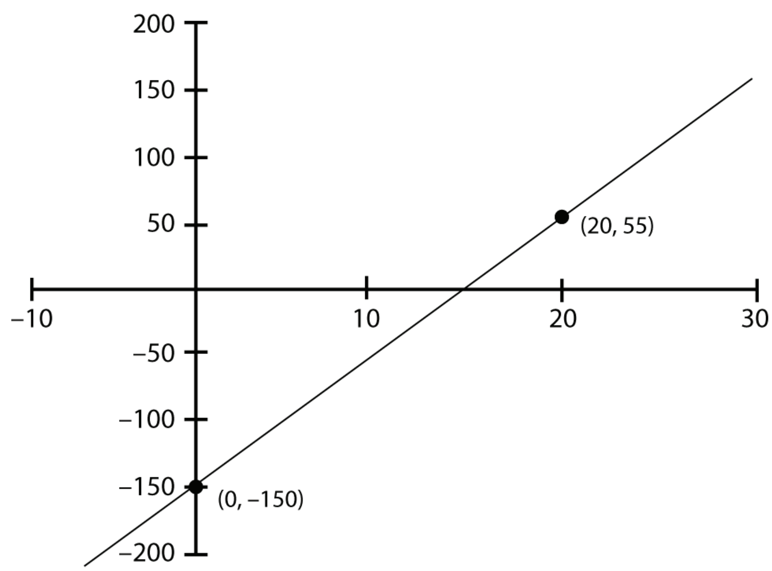
Range: The range is the set of all real numbers y, $0 \leq y \leq 12$, by the same reasoning.

Note to the teacher: This is an example of a ***piecewise-defined function****, which is not formally defined until precalculus. At this point, we are asking students only to determine the function rule for each phase.*

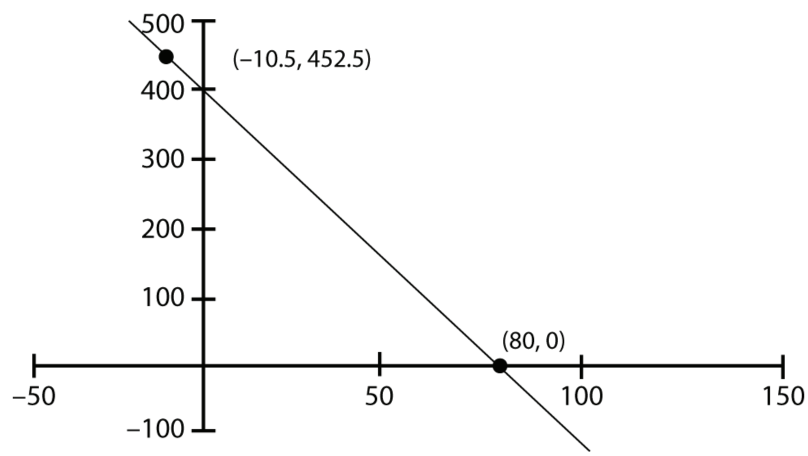
Create a Situation

Create and describe in detail a situation that each of the following graphs could represent. You may use the identified points in your situation, or you may use other points that would lie on that particular graph.

Graph A



Graph B



Notes

CCSS Content Task

(8.F) **Use functions to model relationships between quantities.**

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

(F-IF) **Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

CCSS Additional Teacher Content

(8.F) **Define, evaluate, and compare functions.**

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = *s*2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Scaffolding Questions

- What type of function do these graphs represent?
- What are the constants in the functions?
- Are the functions increasing or decreasing?
- How can you use this information to describe a situation each function might represent?

Sample Solutions

Create and describe in detail a situation that each of the following graphs could represent. You may use the identified points in your situation, or you may use other points that would lie on that particular graph.

Graph A

Using the two points given in the graph, students can determine that the slope is 10.25. The *y*-intercept is given. The following financial situation could be modeled by this graph and function.

> You decide to start a lawn-mowing business. You borrow \$150 from your dad to buy a new mower. You charge \$10.25 for each lawn you mow. The graph represents your balance after you have mowed *x* lawns. You will make a profit once the *y*-value is positive. You are in the red until you mow the 15th lawn, since your break-even point (*x*-intercept) lies between 14 and 15. After that you show a profit, since the *y*-value is positive when you have mowed 15 or more lawns.

Although it is not required, students may also write a function rule, which is $y = 10.25x - 150$. The function rule implies that the *y*-value is increased by 10.25 for every unit change in the *x*-value, and the starting amount is –150.

Graph B

Using the two points given in the graph, students can determine that the slope is –5 and the *y*-intercept is 400. The following situation could be modeled by this graph and function.

This could represent the altitude of a skydiver whose parachute opens at 400 meters. The skydiver is gently drifting to a landing at a rate of 5 meters per second. The graph for this situation would lie only in the first quadrant region. The y-intercept, (0, 400), represents the opening of the parachute. The x-intercept, (80, 0), represents the number of seconds it takes the skydiver to land. The graph could be extended to include the second quadrant region by assuming that $x = 0$ is the point at which the skydiver is first sighted by someone on the ground and that she opened her parachute before that.

Although it is not required, students may also write a function rule, which is $y = 400 - 5x$. The starting value is 400, and the y-value is decreased by 5 units for every increase of 1 in the x-value.

Extension Questions

- What would happen to Graph A if the function were $y = 10.25x - 129.5$? How would this change the situation you described?

  The graph would be a line with the same slope but with a different y-intercept, (0, –129.50). In the situation described above, it could mean that you need to mow fewer yards to break even because you borrowed only $129.50 from your dad.

- What would happen to Graph B if the function were $y = 400 - 4x$? How would this change the situation you described?

  The graph would be a line with the same y-intercept. It would not be as steep, since the slope is –4 instead of –5. (The speed is the absolute value of the slope, so the original situation had a faster rate of change. The skydiver fell faster in the first situation.) The x-intercept would change from (80, 0) to (100, 0).

  In this new situation, it would mean that the skydiver drifts to her landing at 4 meters per second and lands in 100 seconds.

- What would Graphs A and B look like if you reflected the original graphs over the x-axis? How would doing

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.*

Standards for Mathematical Practice

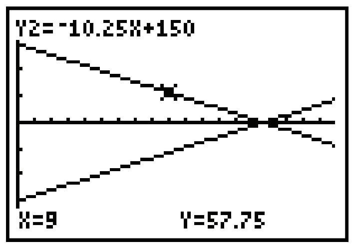
2. Reason abstractly and quantitatively.

this change the functions describing the graphs? How would it change the situations you chose to represent the graphs?

The function for Graph A would become y = –10.25x + 150, since reflecting over the x-axis is the same as multiplying the expression 10.25x – 150 by –1. This is a decreasing linear function. It could no longer represent a "money-earned" situation, but it could represent a "money spent out of $150" situation. For example, Jack has $150 in his savings account. He withdraws $10.25 each week. If he does not add any money to the account, y represents the amount of money in the savings account at x weeks.

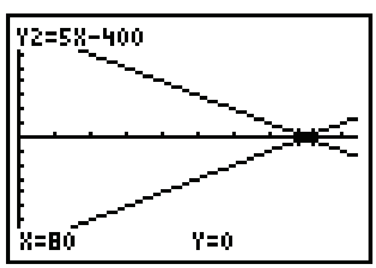
y = 10.25x – 150

y = –10.25x + 150



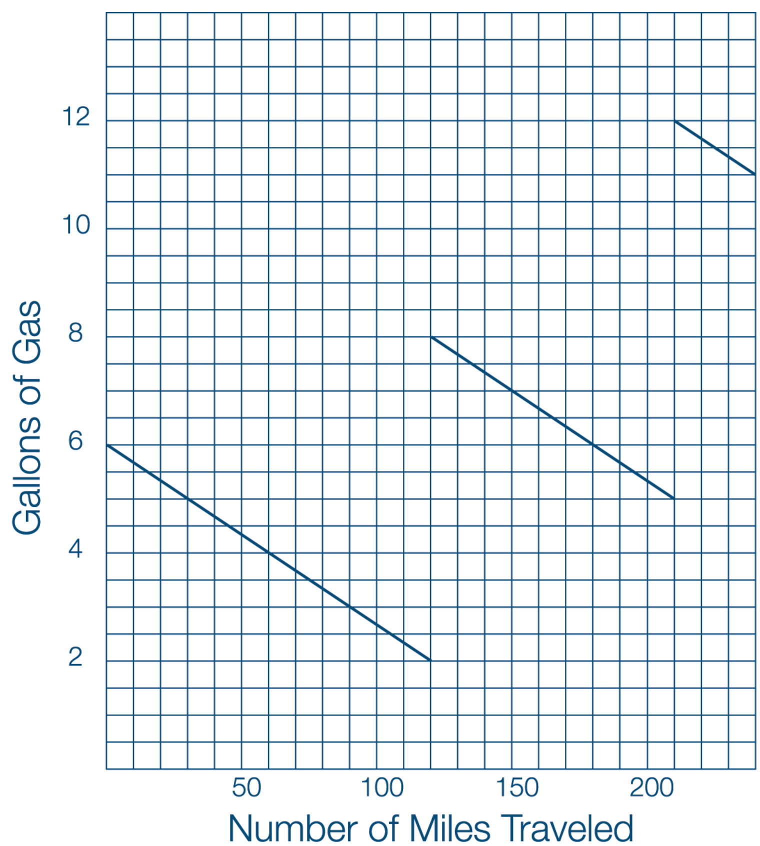
Similarly, the function for Graph B would become y = 5x – 400. The graph would have a negative y-intercept, (0, –400). This is an increasing function that starts at a negative value and could not represent the skydiver's altitude as she drifts to her landing because the altitude at time zero cannot be negative.

Instead, we must think of a situation that begins with a negative value. For example, Lance borrows $400 from his sister and pays her back at the rate of $5 per week. If he continues to pay her at this constant rate, y represents the amount of money he owes her, and x represents the number of weeks he has made payments. The x-intercept is 80; this means that after 80 weeks, Lance owes his sister 0 dollars.



Gas Tank

The following graph shows how the amount of gas in a car's tank varied as a function of the number of miles traveled on a trip. Write a paragraph interpreting the graph for this situation. Include in your description an interpretation of the slopes of the segments.



Notes

CCSS Content Task

(8.F) **Use functions to model relationships between quantities.**

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

(F-IF) **Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

CCSS Additional Teacher Content

(6.EE) **Represent and analyze quantitative relationships between dependent and independent variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation

Scaffolding Questions

- How many phases do you see in the graph?
- How does the graph behave in each phase? What does this mean in the situation?
- How does the graph behave between phases? What does this mean in the situation?
- How does the amount of gas in the tank vary during the first 100 miles of the trip? During the next 120 miles? During the last 40 miles?

Sample Solutions

The graph shows how the amount of gas in a car's tank varied as a function of the number of miles traveled on a trip. Write a paragraph interpreting the graph for this situation. Include in your description an interpretation of the slopes of the segments.

The gas tank starts with 6 gallons of gas. For the first 120 miles, the gas level drops at a steady rate to 2 gallons. At 120 miles, the number of gallons jumps to 8, which suggests that the driver stopped to get gas. During the next 90 miles (from mile 120 to mile 210), the gas level drops steadily to 5 gallons. Again, at 210 miles, the number of gallons jumps suddenly, this time to 12 gallons. Then it drops steadily during the next 30 miles.

Some students may point out that the capacity of the tank is at least 12 gallons, since that is the maximum y-value we see. Thus, at the beginning of the trip, the tank was not full, and on the first refill, the tank was not filled to capacity.

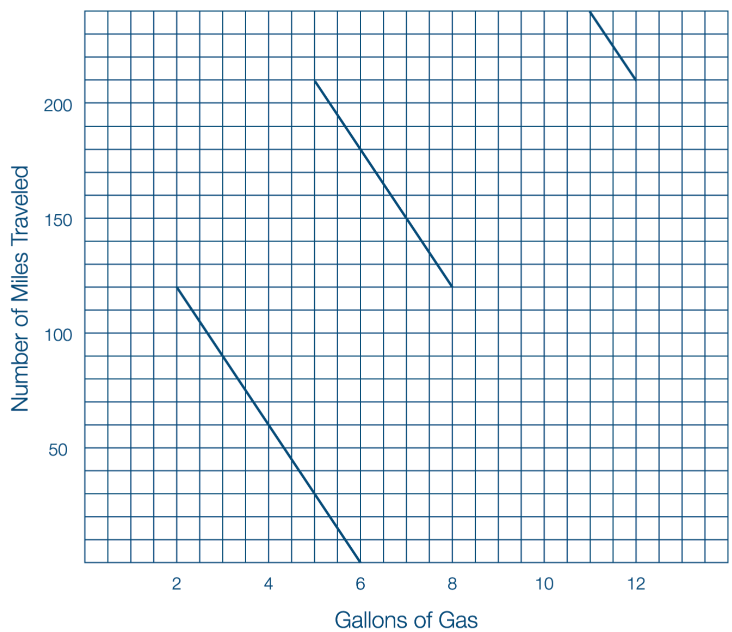
The rate of change in gas in all three phases is 1 gallon used per 30 miles (slope of $-\frac{1}{30}$), so gas consumption (gallons/mile) is occurring at a steady rate. In other words, the car uses $\frac{1}{30}$ of a gallon per mile. Some students may intuitively interpret the rate in this situation as 30 miles per gallon. While it is true that the car gets 30 miles per gallon, that rate does not match the graph and its negative slope.

Extension Questions

- Is it possible for this car to travel 400 miles on a single tank of gas?

  Answers will vary. It may be possible. The tank holds at least 12 gallons and is capable of getting 30 miles per gallon, so it could go at least 12 x 30 = 360 miles. For this car to get 400 miles on one tank, either the tank holds more than 12 gallons or the car uses less gas per gallon (possibly by driving at a slower speed).

- Create a new graph that shows the result of switching the independent and dependent variables of the original graph.



- What does the resulting rate of change (slope) in each phase now represent?

  The rate of change is miles traveled per gallon and is a decrease of 30 miles per gallon in each phase.

to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

(8.F) **Define, evaluate, and compare functions.**

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = *s*2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

6. Attend to precision.

Greetings

The school choir purchased customized cards from a company that charges a \$100 set-up fee and \$2 per box of cards. The choir members will sell the cards for \$3 per box.

The function describing the choir's profit, p dollars, for selling x boxes of cards is $p = 3x - (100 + 2x)$.

1. What do the expressions $3x$ and $100 + 2x$ mean in this situation?

2. How much money will the choir make if the members sell 200 boxes? Show your strategy.

3. How many boxes must the choir sell to make a \$200 profit? Explain how you found your answer.

4. How many boxes must the choir sell to make a \$500 profit? Use a different strategy than the one you used in number 3.

5. How many boxes must the choir sell to break even?

6. The choir will not consider this fundraising project unless they can raise at least \$1,000. Write and solve an inequality that helps them determine if they should take on this project.

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*

   a. Interpret parts of an

Scaffolding Questions

- What does the word *profit* mean?
- What does the 3 in the expression $3x$ represent?
- What does x represent?
- What does p represent?
- Why are parentheses used in the function rule?
- How can you use the distributive property to simplify the expression?
- Which variable are you given in question 2?
- Which variable represents $200 in question 3?
- What does it mean to *break even*?
- Describe how you might use a table to answer question 3.
- Describe how you might use a graph to answer question 3.

Sample Solutions

1. What do the expressions $3x$ and $100 + 2x$ mean in this situation?

   The expression $3x$ represents the total revenue, which is the amount in dollars collected from the sale of x boxes. The $(100 + 2x)$ represents the total cost: The choir has to pay $100 plus $2 per box.

2. How much money will the choir make if the members sell 200 boxes? Show your strategy.

   If the choir sells 200 boxes, you must evaluate the function for $x = 200$.

   $p = 3x - (100 + 2x)$

   $p = 3(200) - (100 + 2(200))$

   $p = 600 - (100 + 400)$

   $p = 600 - 500$

   $p = 100$

   They would make a profit of $100.

3. How many boxes must the choir sell to make a $200 profit? Explain how you found your answer.

   One possible solution:

   Generate a table that shows the number of boxes and the amount of profit made. Use the table to determine the number of boxes that would make a $200 profit.

Number of Boxes	Profit in Dollars
0	–100
100	0
200	100
300	200
400	300
500	400
600	500

   The choir must sell 300 boxes to make a $200 profit.

4. How many boxes must the choir sell to make a $500 profit? Use a different strategy than the one you used in number 3.

   One possible solution:

   The symbolic method can be used to determine how many boxes the choir must sell to make a $500 profit.

   Simplify the rule.

   $p = 3x - (100 + 2x)$

   $p = 3x - 100 - 2x$

   $p = x - 100$

   Substitute 500 for p.

   $500 = x - 100$

   $600 = x$

   The choir must sell 600 boxes to make $500.

expression, such as terms, factors, and coefficients.

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

(A-REI) **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

CCSS Additional Teacher Content

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

8. Analyze and solve pairs of simultaneous linear equations.

c.Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

7. Look for and make use of structure.

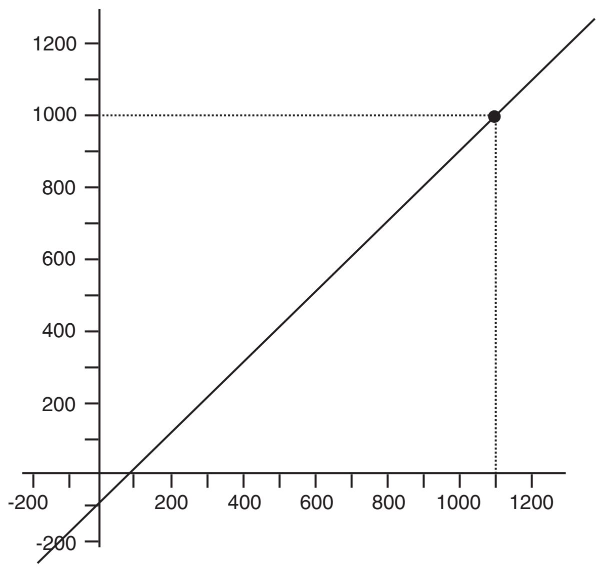
5. How many boxes must the choir sell to break even?

   To *break even* means that the cost equals the revenue, or that the profit is 0.

   One solution is to use the table from the solution for question 3 above to find where $y = 0$ (at $x = 100$). The choir must sell 100 boxes to break even.

   Another solution is to graph the function rule:

   $p = 3x - (100 + 2x)$, or $p = x - 100$



   The break-even point can be seen where the graph intersects the *x*-axis—that is, at 100 boxes. Any boxes sold after the first 100 would generate a profit (where the graph is above the *x*-axis).

6. The choir will not consider this fundraising project unless they can raise at least \$1,000. Write and solve an inequality that helps them determine if they should take on this project.

   If the choir wants to make at least \$1,000, then the profit must be greater than or equal to \$1,000. One approach is to set up an inequality.

$$\begin{aligned} p &\geq 1{,}000 \\ x - 100 &\geq 1{,}000 \\ x &\geq 1{,}100 \end{aligned}$$

   The choir must sell at least 1,100 boxes of cards. If they feel they cannot sell at least 1,100 boxes, they should not undertake this project.

   Another approach is to examine the graph. The graphs of $y = 3x - (100 + 2x)$ and $y = 1{,}000$ intersect at the point (1,100, 1,000). That means that when the choir sells 1,100 boxes, the profit is \$1,000. The graph of the profit is above the graph of $y = 1{,}000$ for values of x greater than 1,100. The choir must sell at least 1,100 boxes to make a profit of at least \$1,000.

Extension Questions

- What would happen in this situation if the \$100 set-up fee were waived?

  The choir would have to pay less. The profit would be represented by p = 3x – 2x, or p = x. The choir would make \$1.00 per box. Now the y-intercept is zero. The rate of change is still \$1.00 per box.

- For another situation, the choir's profit is represented by $p = 3x - (30 + 2.50x)$. Describe the cost and selling process for this situation.

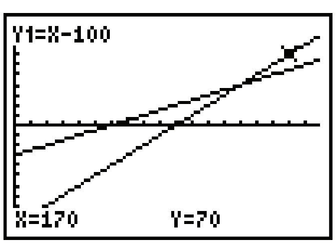
  Profit is revenue minus cost. The expression 3x means that the choir is charging \$3 per box. The cost is represented by 30 + 2.50x. The choir will have to pay a set-up fee of \$30 plus \$2.50 per box.

- Under what conditions is the second situation better than the first?

  One strategy is to determine when the two are equal in value.

$$\begin{aligned} 3x - (30 + 2.50x) &= 3x - (100 + 2x) \\ 0.5x - 30 &= x - 100 \\ -0.5x &= -70 \\ x &= 140 \end{aligned}$$

  Another strategy is to examine the graph to determine which function has the greater value after x = 140. Enter both equations into a graphing calculator, Y1 = x – 100 and Y2 = 0.5x – 30. These graphs intersect at x = 140, meaning that for 140 boxes, cost and profit are the same.

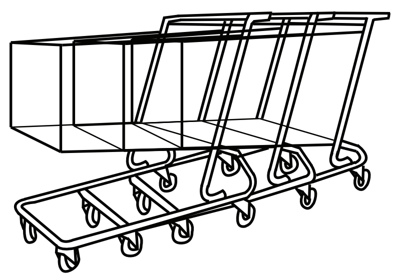


  However, when x is greater than 140, the function y = x – 100 has the greater value. For example, when x equals 170, the graph of the function Y1 = x = 100 is above the graph of Y2 = 0.5x – 30. Therefore, the profit is greater in the first situation for 140 or more boxes.

Grocery Carts

Randy must fit shopping carts into an area that has a length of 82 feet and a width of one shopping cart. He made some measurements necessary for his computations. The table shows the length of a set of shopping carts as they are nested together.

Number of (Nested) Shopping Carts	Length in Inches
1	37.5
8	116.25



Three nested shopping carts are shown.

Randy has decided to use an algebraic expression to determine the length of nested shopping carts.

1. Determine a function rule for the length, in inches, of a set of nested carts in terms of the number of nested shopping carts.
2. How do the numbers in the function rule relate to the physical shopping carts?
3. What is the length of 50 nested shopping carts?
4. How many carts fit into an area with a length of 82 feet and a width of one shopping cart? Use algebraic methods and verify your solution using a table or a graph.

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or

Scaffolding Questions

- What do you know about the shopping carts?
- How long is one cart?
- What is the total length of two nested carts?
- If one cart is 37.5 inches long, how could you find the additional length for each nested cart?
- Will the model for these data be linear? Why or why not?
- How can you compute the rate of change for the situation?
- Complete this new table with the missing values.

Number of (Nested) Shopping Carts	Process	Length in Inches
1	37.5	
2	37.5 +	
3	37.5 +	
4	37.5 +	
5	37.5 +	
n		

Sample Solutions

1. Determine a function rule for the length, in inches, of a set of nested carts in terms of the number of nested shopping carts.

   The length for one cart is 37.5 inches. The rate of change is 78.75 inches for 7 carts or 11.25 inches for 1 cart.

The total length is 37.5 plus 11.25 for every additional shopping cart.

$$L = 37.5 + 11.25(n - 1)$$

or

$$L = 26.25 + 11.25n$$

where n is the number of carts and L is the length of the set of carts.

The function may also be represented by a table or a graph.

2. How do the numbers in the function rule relate to the physical carts?

The 26.25 inches represents the nested length of the carts; that is, it is the length of the cart that slides into the cart in front of it each time. The 11.25 inches is the amount that hangs out for each new cart.

Number of Shopping Carts	Process	Length in Inches
1	26.25 + 11.25(1)	37.5
2	26.25 + 11.25(2)	48.75
3	26.25 + 11.25(3)	60
4	26.25 + 11.25(4)	71.25
5	26.25 + 11.25(5)	82.5
n	$26.25 + 11.25n$	

from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

CCSS Additional Teacher Content

(F-IF) **Interpret functions that arise in applications in terms of the context**

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

1.Distinguish between situations that can be modeled with linear functions and with exponential functions.*

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

4. Model with mathematics.

Graphing calculator table:

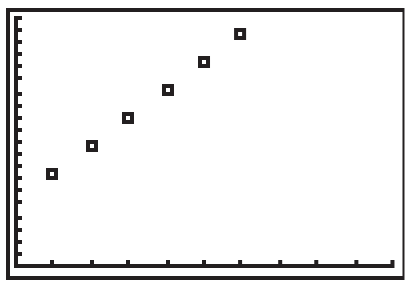
Enter Y1 = 26.25 + 11.25*x*

L1	L2	L3 1
1	37.5	------
2	48.75	
3	60	
4	71.25	
5	82.5	
6	93.75	
7	105	

L1(7)=7

Graph:

WINDOW
Xmin=0
Xmax=10
Xscl=1
Ymin=0
Ymax=100
Yscl=5
Xres=



3. What is the length of 50 nested shopping carts?

When there are 50 carts, substitute 50 for *n* in the formula:

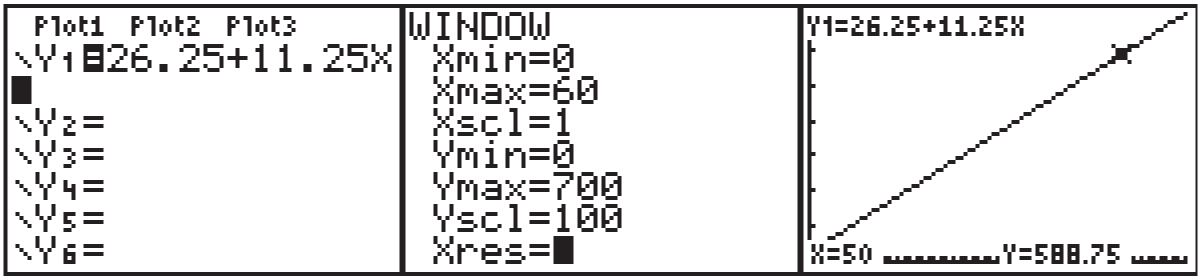
$$L = 26.25 + 11.25(50) = 588.75 \text{ inches}$$

This value can be determined using the calculator table.

X	Y1	
50	588.75	
60	701.25	
70	813.75	
80	926.25	
90	1038.8	
100	1151.3	
110	1263.8	

X=50

This value can also be determined using the calculator graph.



4. How many carts fit into an area with a length of 82 feet and a width of one shopping cart? Use algebraic methods and verify your solution using a table or a graph.

   82 feet must be converted to inches.

   $82 \bullet \frac{12}{1} = 984$ inches

   An equation may be used to determine when L is 984.

   $L = 26.25 + 11.25n$

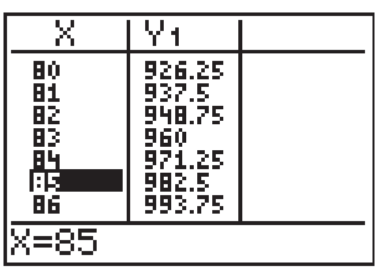
   $984 = 26.25 + 11.25n$

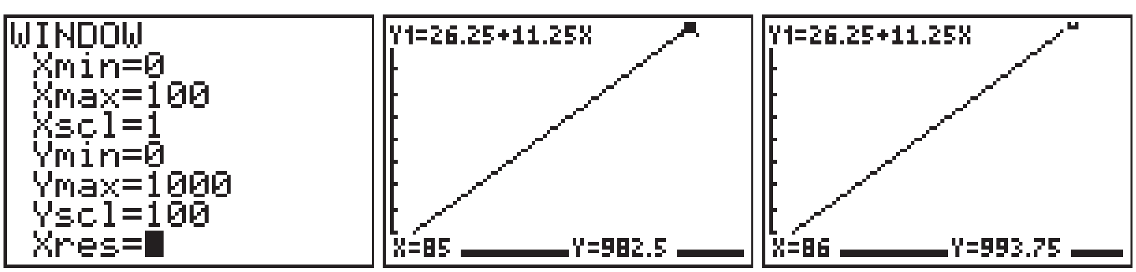
   $957.75 = 11.25n$

   $n = 85.1$, where n represents the number of carts

   There cannot be a fractional number of carts, so the number of carts that fit into 82 feet is 85 carts; 86 carts are longer than 82 feet.

   Verify this answer using the table:





Extension Questions

- If the function rule is $L = 32 + 11.25n$ for a different set of shopping carts, what is the same about the two sets of carts?

  The portion that is added for each new cart is the same because the rate, 11.25, has not changed. However, the y-intercept has changed, so the part that is nested into the rest of the carts is not the same.

- What is a reasonable domain for the function rule representing this different set of carts?

  The function $L = 32 + 11.25n$ is a linear function. The domain of the function is all real numbers.

- What is a reasonable domain for the problem situation?

  The domain for the problem situation represents the number of carts and must be the set of positive integers. However, the domain is determined by the physical and logistical constraints of the situation, such as the available storage space and customer capacity.

- What is the rate of change for this situation.

  The rate of change is 11.25 inches for every one cart.

- How does this change affect the graph?

  The graph has a different y-intercept, but the same slope. The lines are parallel.

- If the function rule is $L = 26.25 + 14n$ for a different set of carts, what is the same about the two sets of carts?

  The portion that is added for each new cart has changed because the rate has changed (from 11.25 to 14). However, the y-intercept has not changed, so the part that is nested into the rest of the carts remains the same.

- How does this change affect the graph?

  The graph has the same y-intercept, but different slopes. The lines are not parallel, but intersect at the point (0, 26.25).

Hull Pressure

When a submarine descends into the ocean, the pressure on its hull increases in increments as given in the following table. (Pressure is measured in kilograms per square centimeter, and depth is measured in meters.)

Depth (m)	0	300	600	900	1,200	1,500
Pressure (kg/cm^2)	0	32	64	96	128	160

1. Describe verbally and symbolically a linear function that relates the depth of the submarine and the pressure on its hull.
2. How does this situation restrict the domain and range of the function?
3. How much pressure will be on the submarine's hull when it is at a depth of 1,575 meters? Describe your solution strategy.
4. If the pressure on the submarine's hull is 240 kg/cm^2, what is the depth of the submarine? How do you know?

Notes

CCSS Content Task

(6.EE) **Reason about and solve one-variable equations and inequalities.**

7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding

Scaffolding Questions

- How does the pressure change as the depth of the submarine increases?
- What is the initial pressure on the submarine's hull?
- What is the dependent variable in this situation?
- What is the independent variable in this situation?
- What is the rate of change in the pressure?
- How will you find the pressure for a given depth?
- How will you find the depth for a given pressure?

Sample Solutions

1. Describe verbally and symbolically a linear function that relates the depth of the submarine and the pressure on its hull.

   When the submarine is at the ocean's surface, the pressure on its hull is 0 kg/cm^2.

   For every 300 meters the submarine dives, the pressure on its hull increases by 32 kg/cm^2.

   A linear function with y-intercept at 0 and slope $m = \frac{32}{300} = \frac{8}{75}$ represents the situation—that is, $p = \frac{8}{75}d$, where p is the pressure in kg/cm^2 and d represents the depth in meters.

2. How does this situation restrict the domain and range of the function?

   While the mathematical domain and range for this function are both the set of all real numbers, the situation restricts the domain to the real numbers from 0 to the maximum depth the submarine can dive. The situation restricts the range to the real numbers from 0 to the maximum pressure the submarine's hull can withstand. This depends on the construction and size of the submarine.

3. How much pressure will be on the submarine's hull when it is at a depth of 1,575 meters? Describe your solution strategy.

If the submarine dives to 1,575 meters, then d = 1,575 and $p = \frac{8}{75}$.

$d = \frac{8}{75}(1{,}575) = 168$

The pressure on the submarine's hull is 168 kg/cm^2.

4. If the pressure on the submarine's hull is 240 kg/cm^2, what is the depth of the submarine? How do you know?

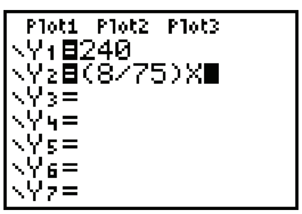
If the pressure on the submarine's hull is 240 kg/cm^2, then p = 240 and the following equation can be solved for d:

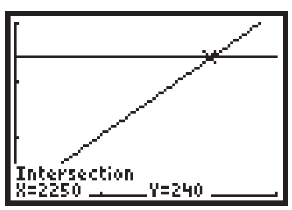
$240 = \frac{8}{75}d$

$d = 240 \bullet \frac{75}{8} = 2{,}250$

The submarine's depth is 2,250 meters.

The problem could also be solved by finding the intersection of the graphs of y = 240 and $y = \frac{8}{75}x$.





The value of x when y = 240 is 2,250.

Extension Question

- Is there a proportional relationship between hull pressure and depth? Explain how you know whether the relationship is proportional.

  The graph of the function is a straight line that contains the point (0, 0). Therefore, there is a proportional relationship between hull pressure and depth.

expressions using the distributive property and collecting like terms.

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(A-REI) **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-IF) **Interpret functions that arise in applications in terms of the context**

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS Additional Teacher Content

(6.EE) **Represent and analyze quantitative relationships between dependent and independent variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

(7.RP) **Analyze proportional relationships and use them to solve real-world and mathematical problems.**

2. Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

4. Model with mathematics.

Notes

Math-a-Thon

Catrina is participating in the school math-a-thon to raise money for the end-of-year field trip. Her mother is donating $25 to get her started. She will also receive 75 cents for every problem she answers correctly.

1. What is the function rule for this situation? Explain the meaning of each constant and variable in your rule.
2. Catrina's grandmother gives her an extra $20 to add to her field trip money. How does this change the previous situation's rule, graph, and table?
3. What part of the situation would you change to produce a less steep slope? A steeper slope? Explain how you know.

Notes

CCSS Content Task

(8.F) **Define, evaluate, and compare functions.**

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*
   a. Interpret parts of an expression, such as terms, factors, and coefficients.

(A-CED) **Create equations that describe numbers or relationships**

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Scaffolding Questions

- How much money will Catrina raise in the math-a-thon if she works 20 problems correctly?
- What are the constants in this situation?
- What are the variables in this problem?
- What type of graph do you think this situation will produce?
- What role does the \$25 play in the graph of this situation?
- What does adding \$20 do to the graph of the situation?
- What is the rate of change for the original situation?
- What is the rate of change for the situation described in number 2?

Sample Solutions

1. What is the function rule for this situation? Explain the meaning of each constant and variable in your rule.

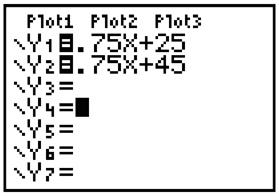
   The amount of donation is \$25 plus \$0.75 times the number of problems Catrina answers correctly. The function rule is $d = 0.75p + 25.00$, where d represents the total amount of donation and p represents the number of problems Catrina answers correctly.

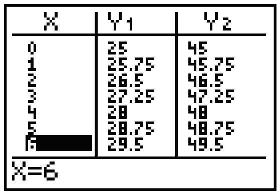
   The \$0.75 is the amount Catrina earns for each problem she solves correctly. The \$25 is the amount Catrina gets from her mother regardless of the number of problems she solves.

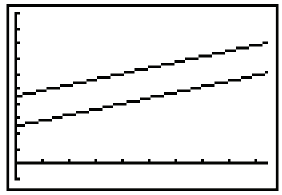
2. Catrina's grandmother gives her an extra \$20 to add to her field trip money. How does this change the previous situation's rule, graph, and table?

   The \$25 in the first function rule changes to \$45, because Catrina now starts with \$25 plus \$20. The new rule is $d = 0.75p + 45$.

   The table now shows that when x is 0, y is 45 instead of 25. y still increases by 0.75 for every problem solved correctly. The two graphs are parallel lines, one starting at (0, 25) and the other starting at (0, 45).







3. What part of the situation would you change to produce a less steep slope? A steeper slope? Explain how you know.

   The amount of money Catrina receives per correct problem affects the rate of change and thus the slope of the line.

   To change the slope in this function rule, we would have to change the amount of money Catrina receives per correct problem. Anything more than \$0.75 produces a line with steeper slope, and anything less than \$0.75 produces a line with a less steep slope.

Extension Questions

- How do the domain of the function rule and the domain of the problem situation compare?

  The domain for the function that models the situation is all real numbers. However, in the problem situation, the number of problems must be a whole number. The number of problems in the competition is the maximum number Catrina could answer correctly, so the domain of the problem situation is a subset of the set of whole numbers.

- How do the graph of the function rule and the graph of the situation compare?

  The graph of the function is a straight line, but the graph of the problem situation is a set of points on a straight line in the first quadrant.

(F-IF) **Analyze functions using different representations**
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

Notes

CCSS Additional Teacher Content

(F-IF) **Interpret functions that arise in applications in terms of the context**

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

1.Distinguish between situations that can be modeled with linear functions and with exponential functions.*

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

4. Model with mathematics.

- Write another scenario that produces a similar function rule, graph, and table.

  Johnny has a basket with 20 apples and starts picking apples at a rate of 5 apples per minute. How many apples does he have in 10 minutes?

- Jackie did not receive a starting donation. Can she still collect as much money as Catrina? Explain your answer.

  Yes, Jackie can answer more problems correctly than Catrina does, and/or she can collect more money per problem.

Shopping

Celeste is shopping for two pairs of shoes and some earrings. She can spend a maximum of $100. The shoes Celeste wants to buy cost $24.99 a pair. Earrings cost $12.99 a pair. The sales tax is 8% of the total price of Celeste's purchases.

1. Write an inequality to identify how many pairs of earrings Celeste can purchase.

2. What is the greatest number of pairs of earrings Celeste can buy? Explain your answer.

3. How many pairs of earrings could Celeste purchase if she finds the shoes on sale for $19.99?

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Scaffolding Questions

- What are you trying to find out? What do you know?
- What does the variable represent in the situation? What is changing and what stays the same?
- How will you show the tax in your inequality?

Sample Solutions

1. Write an inequality to identify how many pairs of earrings Celeste can purchase.

   Let x = the number of pairs of earrings Celeste can buy, \$24.99 = the cost of one pair of shoes, and \$12.99 = the cost of one pair of earrings.

   The cost of 2 pairs of shoes at \$24.99 each + x pairs of earrings at \$12.99 each may be represented by $2(24.99) + 12.99x$.

   The tax of 8% on the sale is represented by $0.08[2(24.99) + 12.99x]$.

   The total cost of the purchases plus the tax cannot exceed \$100. The following sample inequality describes the restriction:

   $$2(24.99) + 12.99x + 0.08\,[2(24.99) + 12.99x] \leq 100.00$$

   Some students may use 1.08 for the price including tax and represent the situation using this inquality:

   $$1.08[2(24.99) + 12.99x] \leq 100$$

2. What is the greatest number of pairs of earrings Celeste can buy? Explain your answer.

   Use the distributive property to simplify before solving and round to the nearest hundredth:

   $$\begin{aligned} 49.98 + 12.99x + 4.00 + 1.04x &\leq 100.00 \\ 53.98 + 14.03x &\leq 100.00 \\ -53.98 \qquad\qquad &\quad -53.98 \\ \hline 14.03x &\leq 46.02 \\ x &\leq 3.28 \end{aligned}$$

Celeste can buy no more than 3 pairs of earrings. She will have some money left over, but not enough for a fourth pair of earrings.

3. How many pairs of earrings could Celeste purchase if she finds the shoes on sale for $19.99?

   If Celeste finds the shoes on sale for $19.99 a pair, the inequality changes as follows:

$$2(19.99) + 12.99x + 0.08[2(19.99) + 12.99x] \leq 100.00, \text{ or}$$

$$1.08[2(19.99) + 12.99x] \leq 100$$

   Use the distributive property to simplify before solving, and round to the nearest hundredth:

$$\begin{aligned} 39.98 + 12.99x + 3.20 + 1.04x &\leq 100.00 \\ 43.18 + 14.03x &\leq 100.00 \\ -43.18 \qquad\qquad &\quad -43.18 \\ \hline 14.03x &\leq 56.82 \\ x &\leq 4.05 \end{aligned}$$

   If the shoes cost $19.99 per pair, Celeste can buy 4 pairs of earrings and still have money left over.

Extension Questions

- How would your solution change if Celeste found the shoes on sale for 25% off the original price? Show your solution algebraically.

  If Celeste found the shoes on sale for 25% off the original price of $24.99, the new shoe price could be found by starting with the idea that the shoes cost 75% of the original price. Multiplying $24.99 by 0.75 gives you the new shoe price.

  24.99(0 .75) = 18.7425; the shoes would cost $18.74 per pair.

  The new inequality would be:

$$\begin{aligned} 2(18.74) + 12.99x + 0.08[2(18.74) + 12.99x] &\leq 100.00 \\ 37.48 + 12.99x + 3.00 + 1.04x &\leq 100.00 \\ 40.48 + 14.03x &\leq 100.00 \\ -40.48 \qquad\qquad &\quad -40.48 \\ \hline 14.03x &\leq 59.52 \\ x &\leq 4.24 \end{aligned}$$

  Celeste could purchase 4 pairs of earrings and still have money left over.

CCSS Additional Teacher Content

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

4. Model with mathematics.

- How would a 9% tax rate affect how many pairs of earrings Celeste could purchase along with shoes at the original price?

  $2(24.99) + 12.99x + 0.09[2(24.99) + 12.99x] \leq 100.00$

  Use the distributive property to simplify before solving, and round to the nearest hundredth:

$$\begin{aligned} 49.98 + 12.99x + 4.50 + 1.17x &\leq 100.00 \\ 54.48 + 14.16x &\leq 100.00 \\ -54.48 \qquad &\quad -54.48 \\ \hline 14.16x &\leq 45.52 \\ x &\leq 3.21 \end{aligned}$$

  A 9% sales tax rate would cost a little more, but Celeste could still purchase 3 pairs of earrings and have money left over.

- Suppose Celeste wants to have \$20 left. Describe and write your solution algebraically, assuming she finds the shoes on sale for \$19.99.

  For Celeste to have \$20 left, the total amount she can spend has to be reduced by \$20. Rather than having \$100 to spend, she now has \$80. The inequality is:

$$\begin{aligned} 2(19.99) + 12.99x + 0.08[2(19.99) + 12.99x] &\leq 80.00 \\ 39.98 + 12.99x + 3.20 + 1.04x &\leq 80.00 \\ 43.18 + 14.03x &\leq 80.00 \\ -43.18 \qquad &\quad -43.18 \\ \hline 14.03x &\leq 36.82 \\ x &\leq 2.62 \end{aligned}$$

  Celeste can purchase only 2 pairs of earrings if she sets aside \$20.

Sound Travel

Many fishing boats and salvage ships are equipped with sonar to measure sound waves to help them find shipwrecks and large schools of fish. Sound travels through freshwater at about 1,463 meters per second when the water temperature is 15°C. By measuring the time it takes for the sound waves to travel through the water from the boat to a school of fish, it is possible to calculate the distance from the boat to the fish.

1. Write a function rule that models the relationship between the number of seconds it takes the sound wave to return to the boat and the distance from the boat to the fish. Identify your variables.
2. Describe the graph of this function, including its domain and range. Explain how you know whether there is a direct variation between the number of seconds and the distance in meters.
3. Suppose the sound wave returned from the fish to the boat in 0.05 seconds. Estimate the distance to the fish. Justify your answer.
4. If the distance from the boat to the fish is 24,000 meters, how long does it take the sound wave to return from the fish to the boat? Explain your solution.

Notes

CCSS Content Task

(6.EE) **Reason about and solve one-variable equations and inequalities.**

7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

(6.EE) **Represent and analyze quantitative relationships between dependent and independent variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

(7.RP) **Analyze proportional relationships and use them to solve real-world and mathematical problems.**

2. Recognize and represent proportional relationships between quantities.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent

Scaffolding Questions

- If the sound wave returns from the fish to the boat in 1 second, what is the distance from the boat to the school of fish? What if the sound wave returns in 2 seconds?
- What is the relationship between the distance from the boat and the fish at 1 second and at 2 seconds? At 2 seconds and at 3 seconds?
- What does 1,463 mean in the function?

Sample Solutions

1. Write a function rule that models the relationship between the number of seconds it takes the sound wave to return to the boat and the distance from the boat to the fish. Identify your variables.

   Let d = the distance between the boat and the school of fish in meters.

   Let t = the time in seconds it takes for the sound wave to return from the fish to the boat.

   The time it takes for the sound wave to return from the fish to the boat depends on the distance in meters between the boat and the fish. Time is the independent variable, and the distance is the dependent variable.

   The table shows the time (in seconds) that it takes for the sound wave to travel the distance (in meters) in freshwater.

Time (seconds)	Distance (meters)
0	0
1	1,463
2	2,926
3	4,389

   The function rule is linear because the rate of change is constant. The difference in the time in the table is 1 second. The difference in the distance is an increase of 1,463 kilometers for every increase of 1 second. So the function rule is $d = 1{,}463t$.

2. Describe the graph of this function, including its domain and range. Explain how you know whether there is a direct variation between the number of seconds and the distance in meters.

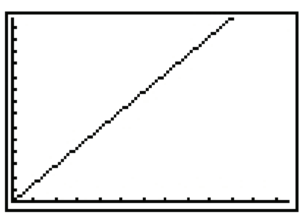
   We can tell from the table that as time increases, distance increases proportionally. We can see that there is a direct variation (proportional relationship) between the distance and the time in seconds because the graph of the function is a straight line that passes through the origin.

   The domain of the function represents the time it takes for the sound wave to return to the boat from the fish. The time in seconds must be greater than 0, or $t > 0$. The range represents the distance between the boat and the fish. The distance must also be greater than 0, or $d > 0$.

   The rate of change in the distance is an increase of 1,463 meters for every second. The slope of the equation is 1,463, representing an increase of 1,463 meters per second. The y-intercept is 0 because at 0 seconds, there is no distance to be recorded.

   The graph of the function is:

```
WINDOW
 Xmin=0
 Xmax=12.6
 Xscl=1
 Ymin=0
 Ymax=14630
 Yscl=1000
 Xres=1
```



3. Suppose the sound wave returned from the fish to the boat in 0.05 seconds. Estimate the distance to the fish. Justify your answer.

   If the sound wave returned from the fish to the boat in 0.05 seconds, the distance from the boat to the fish could be found by substitution into the function as follows:

ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

(8.EE) **Understand the connections between proportional relationships, lines, and linear equations.**

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(A-REI) **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

(F-IF) **Interpret functions that arise in applications in terms of the context**

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

CCSS Additional Teacher Content

(8.F) **Define, evaluate, and compare functions.**

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = *s*2 giving the area of

a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

1.Distinguish between situations that can be modeled with linear functions and with exponential functions.*

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

4. Model with mathematics.

$d = 1{,}463t$

$d = 1{,}463(0.05)$

$d = 73.15$ meters

4. If the distance from the boat to the school of fish is 24,000 meters, how long does it take the sound wave to return from the fish to the boat? Explain your solution.

$$\begin{aligned} d &= 1{,}463t \\ 24{,}000 &= 1{,}463t \\ 16.4 &= t \end{aligned}$$

If the distance to the fish is 24,000 meters, it takes the sound wave approximately 16.4 seconds to return from the fish to the boat.

Extension Questions

- If the time is doubled, is the distance doubled? Justify your answer.

  Because there is a proportional relationship, the distance is doubled if the time is doubled.

  $$\begin{aligned} d &= 1{,}463t \\ 1{,}463(2t) &= 2(1{,}463t) = 2d \end{aligned}$$

- Describe the difference in what is asked in questions 3 and 4 in this activity.

  In question 3, we were given a value from the domain (time) and asked to evaluate the function for distance. In question 4, we were given a function value (distance) from the range and asked to find the corresponding value (time).

- Determine a function rule that expresses time as a function of distance. What type of relationship is this?

  Solve the rule for t.

  $d = 1{,}463t$

  $t = \frac{d}{1{,}463}$, *or* $t = \frac{1}{1{,}463}d$

  This function is also linear and is a proportional relationship.

Taxi Ride

The cab fees in Chicago are $1.40 for the first one-fifth of a mile and 20¢ for each additional one-tenth of a mile as shown in the table.

Miles	Cost in Dollars
$\frac{1}{5}$	1.40
$\frac{1}{5} + \frac{1}{10} = \frac{3}{10}$	1.60
$\frac{4}{10}$	1.80
$\frac{5}{10}$	2.00
$\frac{6}{10}$	2.20
$\frac{7}{10}$	2.40
$\frac{8}{10}$	2.60
$\frac{9}{10}$	2.80
1	3.00

1. How far can you travel for $10.00? Justify the reasonableness of your response.

2. Generate a table to give the cost per mile. How much will you have to pay to get to a restaurant that is 20 miles away from your hotel? Solve using two different methods.

3. If you want to include a 15% tip for the cab driver, how far can you travel for $10.00?

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and

Scaffolding Questions

- What are the variables in this situation?
- Which is the independent variable?
- What kind of relationship is there between the two variables?
- How much would you pay for the first mile?
- How much would you pay for the second mile?
- What is the rate of increase (in dollars) per mile?
- What is the y-intercept?
- What is multiplied by 20¢? Could you use 40¢ instead? How?
- How many tenths are there in $\frac{1}{5}$?

Sample Solutions

1. How far can you travel for \$10.00?

   Use the table to look for patterns, and from the pattern generate the rule for the situation. The pattern shows that the cost increases by 40¢ for every additional fifth of a mile.

Miles	Cost in Dollars
$\frac{1}{5}$	1.40
$\frac{1}{5} + \frac{1}{10} = \frac{3}{10}$	1.60
$\frac{1}{5} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10}$	1.80
$\frac{5}{10}$	2.00
$\frac{6}{10}$	2.20
$\frac{7}{10}$	2.40
$\frac{8}{10}$	2.60
$\frac{9}{10}$	2.80
1	3.00

Determine how many tenths of a mile are left after paying $1.40 for the first fifth of a mile.

Let the number of miles be represented by m.

The number of miles left after the first fifth of a mile is

$m - \frac{1}{5}$.

In each mile, there are 10 tenths.

The number of tenths of a mile left after the first fifth of

a mile is represented by $10(m - \frac{1}{5})$ or $10m - 2$.

The cost is 20¢ for every tenth of a mile or

$10(m - \frac{1}{5})(0.20)$ or $2m - 0.40$.

An expression for the total charge is $\$1.40 + 10(m - \frac{1}{5})(0.20) = \$1.40 + 2m - 0.40 = 2m + 1$.

To determine how many miles can be traveled for $10.00, substitute $10.00 for the cost and solve the inequality.

$10 \geq 1.40 + 10(m - \frac{1}{5})(0.20)$

$10 \geq 1.40 + (10m - 2)(0.20)$

$10 \geq 1.40 + 2m - 0.40$

$10 \geq 2m + 1$

$9 \geq 2m$

$m \leq 4.5$

The greatest number of miles you can travel for $10.00 is 4.5 miles.

2. Generate a table to give the cost per mile. How much will you have to pay to get to a restaurant that is 20 miles away from your hotel? Solve using two different methods.

use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

(A-REI) **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

CCSS Additional Teacher Content

(6.EE) **Represent and analyze quantitative relationships between dependent and independent variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

(F-IF) **Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

Generate a table to find the cost of traveling whole miles.

Miles	Cost in Dollars
1	3.00
2	5.00
3	7.00
4	9.00
5	11.00
6	13.00

The first mile costs $3.00, but each mile thereafter costs $2.00. Continue the table until you find the cost for 20 miles, or use the pattern to determine the rule.

Let c equal the cost in dollars of the taxi ride and m represent the number of miles.

So, $c = 2(m - 1) + 3$.

Substitute the 20 miles for the m and determine the cost.

$c = 2(20 - 1) + 3$

$c = 2(19) + 3$

$c = 38 + 3$

$c = 41$

It costs $41.00 to travel 20 miles.

Another way to solve this problem is to use the rule for the tenth of a mile (from the answer to question 1). Substitute the 20 for the m.

$c = 1.40 + 10(m - \frac{1}{5})(0.20)$

$c = 1.40 + 10(20 - \frac{1}{5})(0.20)$

$c = 1.40 + 10(19.8)(0.20)$

$c = 41.00$

3. If you want to include a 15% tip for the cab driver, how far can you travel for $10.00?

   If the cost is to include a tip of 15%, the computed cost must be multiplied by 1.15 (that is, the original cost + 0.15 of the original cost).

   $10 \geq [1.40 + 10(m - \frac{1}{5})(0.20)]1.15$

   $10 \geq 1.61 + (10m - 2)(0.23)$

   $10 \geq 1.61 + 2.3m - 0.46$

   $10 \geq 2.3m + 1.15$

   $8.85 \geq 2.3m$

   $m \leq 3.847826$

   If you want to leave a 15% tip and travel for less than $10.00, the longest distance you can go is about 3.8 miles.

Extension Questions

- What are reasonable domain and range values for this situation?

  The domain values are every tenth of a mile after the first mile. The range values are $1.40 and every increment of 20¢ after $1.40.

- Does it make any difference which table you use to determine the cost of a taxi ride? Explain.

  It does not make any difference if you are going a whole number of miles, but if you are traveling partial miles, you can calculate your cost more precisely by using the original table.

- If a shuttle service charges a flat fee of $50.00 to any location from the airport, under what circumstances is it more cost effective to take a taxi?

This question is really asking when the taxi would cost less than \$50.00.

Examine the table for the function.

X	Y_1	
24.3	49.6	
24.4	49.8	
24.5	50	
24.6	50.2	
24.7	50.4	
24.8	50.6	
24.9	50.8	

X=24.5

It would be more cost effective to take the taxi to any location that is fewer than 24.5 miles from the airport.

- If taxi rates increase, how would it affect the representation of your data?

  If the rate per tenth of a mile increased, the table entries would be greater and the slope of the graph would be steeper. If the increase also happened in the first fifth of a mile, the y-intercept would also change. If the increase happened only in the first fifth of a mile, only the y-intercept would change.

The Contractor

As a flooring contractor, Lupe sets floor tile for a living. He submits a bid for each new job. When preparing a bid, he measures the area of the floor to be tiled and then figures out how much material he will need. He charges the following prices for materials and labor:

Subflooring:	$1.27 per square foot
Tile:	$6.59 per square foot
Adhesive:	$31.95 per job
Grout:	$55.95 per job
Labor:	$125 base price plus $0.79 per square foot

1. Write an algebraic rule to determine the total cost of the materials and labor for a typical job in terms of the number of square feet. Explain what the numbers and symbols in the rule mean.
2. Make a table and graph to help Lupe see the amount of money he should charge for jobs with various amounts of square footage.
3. For a job tiling an area of 550 square feet, what is the amount of the bid, based on the materials listed above?

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial

Scaffolding Questions

- What are the constants in this problem situation?
- What are the variables?
- Make a list of all the charges Lupe must consider.
- How would he compute the cost of the tile?
- How would he compute the cost of the subflooring?
- How would he compute the labor charges?

Sample Solutions

1. Write an algebraic rule to determine the total cost of the materials and labor for a typical job in terms of the number of square feet. Explain what the numbers and symbols in the rule mean.

   The total cost is equal to the cost of the subflooring *plus* the cost of the tile *plus* the cost of the adhesive *plus* the cost of the grout *plus* the cost of the labor. Fixed materials costs are those for adhesive and grout. Costs for the subflooring and tile depend on the number of square feet to be tiled. Labor costs include both a fixed cost ($125 base price) and a cost per square foot.

   If the area in square feet is represented by the variable x and the total cost of the job is c, then the function rule is

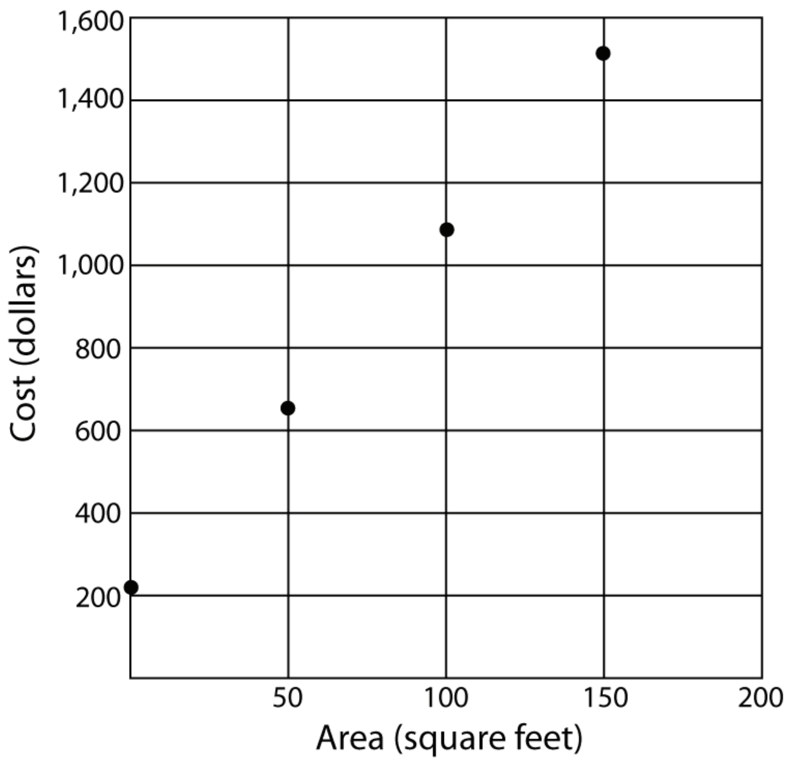
   $$c = 1.27x + 6.59x + 31.95 + 55.95 + 125 + 0.79x$$

   By combining terms, this can be rewritten as $c = 8.65x + 212.90$. $8.65 is the combined cost of the items per square foot of area. $212.90 is the total of the fixed costs that do not depend on area.

2. Make a table and graph to help Lupe see the amount of money he should charge for jobs with various amounts of square footage.

   Use the rule and put the values in a table.

Area (square feet)	Cost (dollars)
0	212.90
50	645.40
100	1,077.90
150	1,510.40



3. For a job tiling an area of 550 square feet, what is the amount of the bid, based on the materials listed above?

   Students may use a table to look for the *x* value of 550.

Area (square feet)	Cost (dollars)
500	4,537.90
550	4,970.40
600	5,402.90

If the area is 550 square feet, the cost is $4,970.40. Students may also use an equation to find the cost: 8.65(550) + 212.90 = 4,970.40.

value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(A-SSE) **Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context.*
   a. Interpret parts of an expression, such as terms, factors, and coefficients.

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-IF) **Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by

hand in simple cases and using technology for more complicated cases.*

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

CCSS Additional Teacher Content

(8.F) **Define, evaluate, and compare functions.**

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values*

and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

(A-REI) **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

(F-IF) **Analyze functions using different representations**

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Standards for Mathematical Practice

4. Model with mathematics.
6. Attend to precision.

Extension Questions

- Suppose Lupe wants to make a 20% profit on each job. Write a new rule to compute how much he should charge his customers, including his profit.

  The new price can be determined by adding 20% of the original cost to the original cost. Using symbols, it is 0.20(8.65x + 212.90) + 8.65x + 212.90, or 1.20(8.65x + 212.90).

- The price of the adhesive increases to \$45.95 per job. How does this affect the cost of the jobs? How does the increase show up in the rule, graph, and table?

  The adhesive cost is part of the fixed amount per job, \$212.90. The fixed costs increase by the difference between the adhesive's new cost and its original cost, \$45.95 – \$31.95, or \$14. The new fixed costs are \$212.90 + \$14, or \$226.90. The table values all increase by this same amount. In the graph, the y-intercept is 226.90.

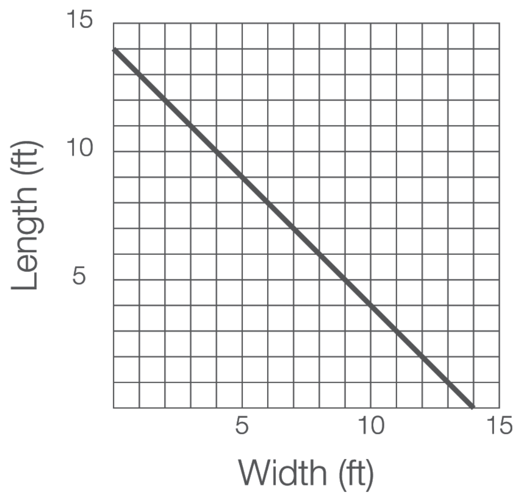
- If the charge for labor changes to \$130 plus \$0.85 per square foot, how does this affect the function that represents the cost of the jobs?

  The price for labor has a fixed amount, and it is also part of the charge per square foot. An increase in the fixed amount of the labor affects the y-intercept. If the fixed amount increases from \$125 to \$130, the y-intercept increases by 5. An increase in the charge per square foot increases the slope of the graph. The increase from \$0.79 to \$0.85 increases the slope of the graph by 0.06.

  The function rule changes from c = 8.65x + 212.90 to c = 8.71x + 217.90.

The Garden

Lance has a certain amount of fencing. He wants to use the fencing to enclose as much of his rectangular garden area as possible. He creates a graph to represent the relationship between the possible lengths and widths of the garden's fencing.



1. Describe verbally and symbolically the relationship between the length and the width of the garden.

2. What are reasonable domain and range values for this function? Create a table to show possible lengths and widths.

3. Explain what the graph tells you about the perimeter of the garden.

Notes

CCSS Content Task

(8.EE) **Understand the connections between proportional relationships, lines, and linear equations.**

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

(A-CED) **Create equations that describe numbers or relationships**

2. Create equations in two or more variables to represent relationships between

Scaffolding Questions

- What type of relationship is described by the graph?
- Name some points on the graph. What does that tell you about the possible dimensions of the garden?
- Define the independent variable and the dependent variable for this problem situation.
- What would the length be if the width were 12 feet?
- What are the restrictions on the length and the width?
- How do you find the perimeter of a rectangle?

Sample Solutions

1. Describe verbally and symbolically the relationship between the length and the width of the garden.

   The graph is a line. The starting value (y-intercept) is 14. The x-intercept is 14.

   As the width of the rectangular area increases, the length decreases. Because Lance is using a fixed amount of fencing, the width and length change at a constant rate. For every 1 foot the width increases, the length must decrease 1 foot. The relationship can be described by the function $l = -1w + 14$.

   Thus, the slope of the line is –1.

   However, there could not be a garden if the width were 0 or 14 feet.

2. What are reasonable domain and range values for this function? Create a table to show possible lengths and widths.

   From the graph you can see that both the length and the width must be positive numbers less than 14. So the domain is $0 < w < 14$, and the range is $0 < l < 14$.

Sample table of possible lengths and widths:

Width (feet)	**Length (feet)**
0	14
2	12
4	10
8	6
10	4
14	0

The points (0, 14) and (14, 0) are not possible solutions in this problem situation. There could not be a garden if either the width or the length was 0.

3. Explain what the graph tells you about the perimeter of the garden.

Notice from the table that the length plus the width must be 14.

$$l + w = 14$$

or

$$l = -1w + 14$$

The perimeter is twice the sum of the width and the length.

$2(l + w) = 28$, or $2l + 2w = 28$

The perimeter or amount of fencing is 28 feet.

quantities; graph equations on coordinate axes with labels and scales.

(F-IF) **Interpret functions that arise in applications in terms of the context**

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Notes

CCSS Additional Teacher Content

(6.EE) **Represent and analyze quantitative relationships between dependent and independent variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

(8.F) **Define, evaluate, and compare functions.**

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

4. Model with mathematics.

Extension Questions

- How is the domain for the function rule different from the domain for the problem situation?

  The domain for the function rule is the set of all real numbers. The domain for the problem situation is the set of all real numbers from 0 to 14.

- How would the graph have been different if the total amount of fencing had been 24 feet?

  Twice the sum of the length and width would have been 24 feet. The sum of the length and the width would have been 12 feet. The x- and y-intercepts would have both been 12.

- What would the graphs have in common if the total amount of fencing had been 24 feet?

  The slope for both graphs would have been –1.

- Describe the relationship between the area and the width of the garden.

  The area is the length times the width. The length is represented by 14 – w.

  $A = (14 - w)w$

The Submarine

A submarine is cruising 195 meters beneath the ocean's surface and begins rising toward the surface at a constant rate of 12 meters per minute.

1. Describe verbally and symbolically a function relating the submarine's position and the amount of time it has been rising.

2. How long does it take the submarine to reach the ocean's surface? Justify your solution using symbols, a table, and a graph.

3. Suppose the submarine started at its original depth (195 meters below sea level) and must reach the ocean's surface 5 minutes sooner than before. Describe how this changes the function and graph of the original situation. What is the new function, and how did you determine it?

Notes

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial

Scaffolding Questions

- What are the constants in the problem? What quantities vary?
- What quantity is the dependent variable? The independent variable?
- Create a table and/or graph to verify your function rule for question 1.
- What kind of function models the situation?
- What is the submarine's depth when it is at surface level?
- What equation will you write and solve?
- What question will solving the equation answer?
- If the submarine must surface 5 minutes sooner, how long will it take to surface?
- What quantity in the original function rule must change?
- Will the submarine rise at the same rate? A slower rate? A faster rate?
- Try different rates in your original function rule. Use tables and/or graphs to estimate the rate at which the submarine needs to rise to reach the surface 5 minutes sooner.
- What equation can you write and solve to determine this new rate?

Sample Solutions

1. Describe verbally and symbolically a function relating the submarine's position and the amount of time it has been rising.

   The submarine's position, D meters below the surface, depends on the time, t minutes, it has been rising. The submarine starts rising from 195 meters below the surface, so its initial position is –195. It is rising toward the surface at 12 meters per minute, which is a constant rate. The function is a linear function because the rate at which the submarine is rising is constant.

The distance is the starting value plus the rate of change multiplied by the number of minutes.

This gives the function: $D = -195 + 12t$.

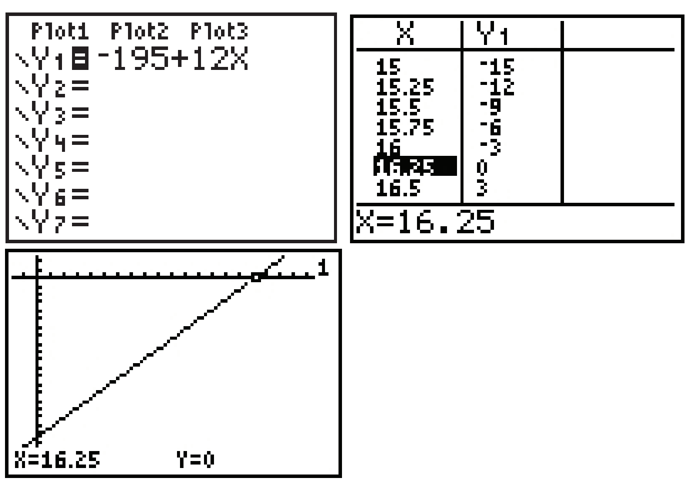
2. How long does it take the submarine to reach the ocean's surface? Justify your solution using symbols, a table, and a graph.

   When the submarine surfaces, its depth below the surface is 0 meters, so

$$\begin{aligned} D &= -195 + 12t \\ 0 &= -195 + 12t \\ -12t &= -195 \\ t &= 16.25 \end{aligned}$$

   So, the submarine takes 16.25 minutes to surface if it starts at 195 meters below the surface and rises at a rate of 12 meters per minute.

   Another approach to answering the question is to use a graphing calculator.



3. Suppose the submarine started at its original depth (195 meters below sea level) and must reach the ocean's surface 5 minutes sooner than before. Describe how this changes the function and graph of the original situation. What is the new function, and how did you determine it?

   If the submarine must surface 5 minutes sooner, it must rise at a faster rate than 12 meters per minute. Before, the submarine took 16.25 minutes to surface. Now the

value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(A-REI) **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

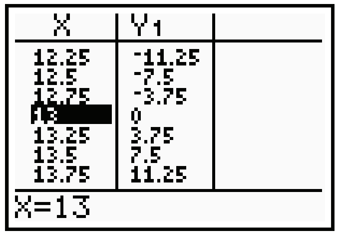
(A-REI) **Solve equations and inequalities in one variable**

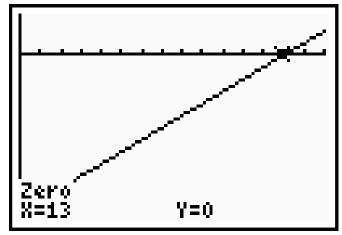
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

submarine must surface in 11.25 minutes.

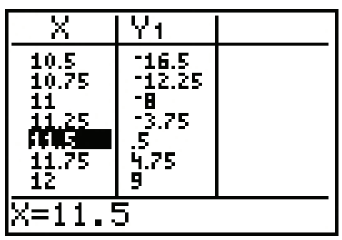
Experiment with different rates (more than 12 meters per minute) by using tables and graphs. If the submarine rises at 15 meters per minute, the equation is $D = -195 + 15t$.

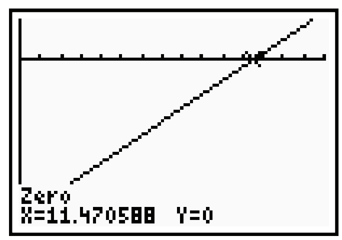
The graph and table show that it takes 13 minutes to reach the surface.



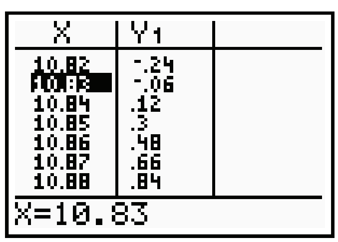


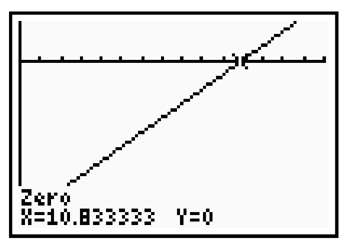
If the submarine rises at 17 meters per minute, the equation is $D = -195 + 17t$. It takes about 11.5 minutes to reach the surface.





If the submarine rises at 18 meters per minute, the equation is $D = -195 + 18t$. It takes about 10.8 minutes to reach the surface.





The submarine needs to surface at a rate between 17 and 18 meters per minute.

Another approach is to let the rate be a variable.

$D = -195 + rt$

(F-IF) **Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

D must be equal to 0 when $t = 11.25$.

$$\begin{aligned} 0 &= -195 + r(11.25) \\ 195 &= 11.25r \\ r &= 17.3 \end{aligned}$$

The rate is 17.3 meters per minute.

Extension Questions

- How do the domain and range of the situation compare with the mathematical domain and range of the function representing this situation? What effect does this have on how you graph the situation?

  The mathematical domain and range of the function are both all real numbers because it is a linear function. The situation restricts the domain to 0 to 16.25 minutes and the range to –195 to 0 meters. Knowing the domain and range of the situation helps determine an appropriate window for the graph on the calculator.

- What are the intercepts of the graph of the function? What information do they give about the situation?

  The y-intercept is (0, –195). The initial depth of the submarine is 195 meters below surface. The x-intercept is (16.25, 0). The submarine takes 16.25 minutes to reach the ocean's surface.

- Suppose the submarine must rise from 195 meters below the surface to the ocean's surface within 10 to 20 minutes. How does this affect the rate at which the submarine rises toward the ocean surface?

  In this case, the constant is a range in time to rise to the surface instead of the rate at which the submarine rises. So let r be the rate, in meters per minute, that the submarine rises and D the depth, in meters, of the submarine.

CCSS Additional Teacher Content

(6.EE) **Represent and analyze quantitative relationships between dependent and independent variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.

$D = rt - 195$

If $d = 0$ at 10 minutes,

$$\begin{aligned} 0 &= r(10) - 195 \\ 10r &= 195 \\ r &= 19.5 \end{aligned}$$

If $d = 0$ at 20 minutes,

$$\begin{aligned} 0 &= r(20) - 195 \\ 20r &= 195 \\ r &= 9.75 \end{aligned}$$

The rate, r meters per minute, must range from 9.75 to 19.5.

- Suppose a student modeled this situation with the function rule $y = 195 - 12t$. What do the variables represent for this rule?

  y represents the distance from the surface to the submarine. The rate of travel is –12 meters per minute because the distance from the surface to the submarine is decreasing at a rate of 12 meters per minute.

  t represents the time it takes to travel from the surface to the submarine.

Speeding Cars

On a racing route, four cars traveling at different speeds all passed through the same checkpoint at the same time. The following data were collected from the performance of the four cars based on miles driven from that checkpoint in terms of hours. Assume that the average speed of each car stayed constant during the interval in which it was collected.

Car A

Hours	Miles
0	0
2	120
3	180
5	300
6	360

Car B

Hours	Miles
0	0
1	75
2	150
3	225
4	300

Car C

Hours	Miles
0	0
5	200
10	400
15	600
20	800

Car D

Hours	Miles
0	0
1	65
2	85
3	105
4	125

1. Using the data in the tables, determine which car was traveling the fastest. How do you know?
2. Which car was traveling the slowest? How do you know?
3. Compare and contrast the tables.
4. If possible, write a function rule to model each car's travel.
5. Create a graph to represent the distance traveled for each car. Compare and contrast the graphs.
6. Compare the domains for the functions and the domains for the problem situation.
7. Do any of the tables represent a direct variation? Explain how you know.

Notes

CCSS Content Task

(7.RP) **Analyze proportional relationships and use them to solve real-world and mathematical problems.**

2. Recognize and represent proportional relationships between quantities.

   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

(8.F) **Define, evaluate, and compare functions.**

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Scaffolding Questions

- How can you use the tables to determine the speed of each car?
- How can you tell whether a car was traveling at a constant rate?
- What are the similarities in the table values?
- What are the differences in the table values?
- How can you tell whether a table represents a linear function?
- How can you tell from a graph that a function is linear?
- What must be true if a set of points represents a direct variation?
- What happens if you do not assume that the average speed of each car stayed constant during each timing interval?

Sample Solutions

1. Using the data in the tables, determine which car was traveling the fastest. How do you know?

   One way to find the speed at which a car traveled is to divide the distance the car traveled by the amount of time it traveled. You can also use the slope formula for any consecutive interval.

Car A:

$$\frac{120-0}{2-0}=60 \qquad \frac{180-0}{3-0}=60 \qquad \frac{300-0}{5-0}=60 \qquad \frac{360-0}{6-0}=60$$

Car A traveled at a constant rate of 60 miles per hour.

Car B:

$$\frac{75-0}{1-0}=75 \qquad \frac{150-0}{2-0}=75 \qquad \frac{225-0}{3-0}=75 \qquad \frac{300-0}{4-0}=75$$

Car B traveled at a constant rate of 75 miles per hour.

Car C:

$\frac{200-0}{5-0}=40$ $\quad \frac{400-0}{10-0}=40$ $\quad \frac{600-0}{15-0}=40$ $\quad \frac{800-0}{20-0}=40$

Car C traveled at a constant rate of 40 miles per hour.

Car D:

$\frac{65-0}{1-0}=65$ $\quad \frac{85-0}{2-0}=42.5$ $\quad \frac{105-0}{3-0}=35$ $\quad \frac{125-0}{4-0}=31.25$

Car D did not travel at a constant rate.

If the rates are examined for the one-hour time intervals, the differences are not the same.

$\frac{65-0}{1-0}=65$ $\quad \frac{85-65}{2-1}=20$ $\quad \frac{105-85}{3-2}=20$ $\quad \frac{125-105}{4-3}=20$

Car D traveled 65 mph for the first hour and then at a constant rate of 20 mph for the next three hours.

Car B traveled the fastest; for every hour, it went 75 miles.

2. Which car was traveling the slowest? How do you know?

   Car C traveled at the slowest constant rate; for every hour, it traveled only 40 miles. Car D, however, traveled at a slower rate after the first hour.

3. Compare and contrast the tables.

   The tables are similar in several ways; all four tables start at (0, 0) and report miles and hours. All the tables show that as the hours increase, the number of miles traveled also increases. The tables are different because the time intervals are not the same in every table. In the table for Car D, there isn't a constant rate throughout the *y*-values.

   Some students may note that the tables for Cars A, B, and C represent linear relationships because they indicate a constant rate of change. The table for Car D does not represent a linear relationship since the car traveled 65 miles in the first hour but only 20 miles for each additional hour.

(A-CED) **Create equations that describe numbers or relationships**

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(F-IF) **Interpret functions that arise in applications in terms of the context**

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

(F-IF) **Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

3. Construct viable arguments and critique the reasoning of others.

4. If possible, write a function rule to model each car's travel.

   The first three tables can be modeled by function rules of the form $y = mx + 0$ (where m is the slope or rate of change) because the starting value is 0.

   Car A: $y = 60x$

   Car B: $y = 75x$

   Car C: $y = 40x$

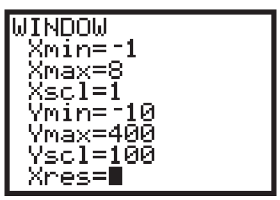
   However, the rule for Car D is different.

   Car D: $\begin{cases} 65x, & 0 \le x \le 1 \\ 20x + 45, & 1 < x \le 4 \end{cases}$

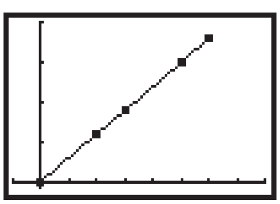
   For values of x between 0 and 1, the rate is 65 mph, but between the values of 1 and 4, the linear model $y = 20x + 45$ fits.

   Note: This is an example of a **piecewise-defined function**, which is not formally defined until Precalculus. At this point, we are suggesting only that students may determine a function rule for each phase.

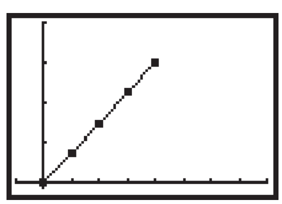
5. Create a graph to represent the distance traveled for each car. Compare and contrast the graphs.



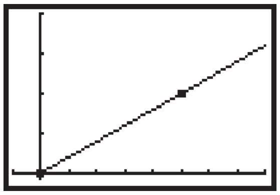
A.



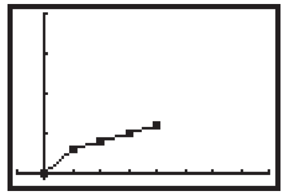
B.



C.



D.



Each graph is a set of points. All graphs start at (0, 0). The lines all have a positive slope. Graph B has the greatest slope because it is the steepest and has the greatest rate of change. Graph D is modeled by a combination of two lines—a piecewise-defined function. Graphs A, B, and C represent linear relationships.

6. Compare the domains for the functions and the domains for the problem situation.

   The domains for the functions are all real numbers. The domains for the problem situation are numbers greater than or equal to 0. The graphs show connected points because the functions are continuous for the values of the number of hours. However, the upper limit on the domain values depends on how long each car traveled.

7. Do any of the tables represent a direct variation? Explain how you know.

   The tables for Cars A, B, and C represent direct variations because there is a constant rate of change and the data contain the point (0, 0). The table for Car D does not represent a direct variation because there is not a constant rate of change.

Extension Questions

- If another car traveled at twice the speed of Car C, how would the table values be affected?

  If the car is traveling at twice the speed of Car C, the rule for the distance traveled as a function of the number of hours would be $y = 80x$.

  If the x-values were the same, the y-values would have been twice the original values of Car C.

- Suppose that another car has the same table values as Car A except that 20 is added to each *y*-value. Describe how this car's motion is the same as or different from that of Car A.

Car A

Hours	Miles
0	0
2	120
3	180
5	300
6	360

New Car

Hours	Miles
0	20
2	140
3	200
5	320
6	380

The new car is traveling at the same rate as Car A—60 mph—although 20 has been added to each y-value.

$\frac{140-20}{2-0}=60$ $\frac{200-20}{3-0}=60$ $\frac{320-20}{5-0}=60$ $\frac{380-20}{6-0}=60$

One possible way to interpret the difference between the two cars' tables is that data for the new car were first collected 20 miles before the car entered the intersection.

- How would the graph of this new car's function be different from the graph of Car A's function?

  The y-intercept of this new graph would be at 20, but the graph would be parallel to the graph of Car A. The situation does not represent direct variation.

Take Your Seat, Please

Eagle Stadium is the home of the state championship Soaring Eagle baseball team. One section in Eagle Stadium contains 20 seats in the first row, 22 in the second row, 24 in the third, and so on, for 25 rows.

1. Write a recursive formula to determine the number of seats, *S*, in row number *n*.

2. Use your graphing calculator to make a scatterplot of the number of seats in terms of the row number.

3. Use your scatterplot, the recursive rule, or the problem situation to determine an explicit function rule for the number of seats for each row number.

4. Compare the constants in your recursive formula to the constants in your explicit function rule. How are they alike in terms of rates of change and initial starting conditions?

5. How does the type of sequence generated by the recursive formula compare to the type of function generated by the explicit function rule? Include any domain restrictions in your description.

Notes

Materials:
Graphing calculator

Connections to the CCSSM

(F-IF) **Understand the concept of a function and use function notation**

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and *translate between the two forms.**

Standards for Mathematical Practice
4. Model with mathematics.

Scaffolding Questions:

- How many seats are added to get the number of seats in the next row?
- How can you use home screen recursion to make a table of values for the 25 rows?
- How could you use the List Editor of your graphing calculator to make a scatterplot?
- What type of function could be used to represent the data? How do you know?
- Is there a constant rate of change? What is it?
- What types of numbers could be used in the domain of the recursive formula?
- What types of numbers could be used in the domain of the explicit function rule?
- What types of numbers could be used in the domain of the explicit function rule for this situation?

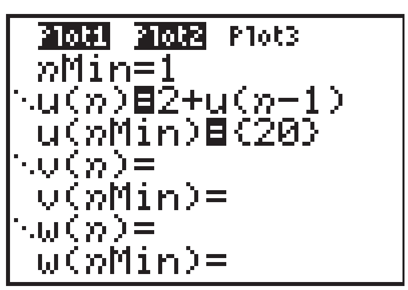
Sample Solutions:

1. Write a recursive formula to determine the number of seats, *S*, in row number *n*.
   $S_n = 2 + S_{n-1}$, $S_1 = 20$

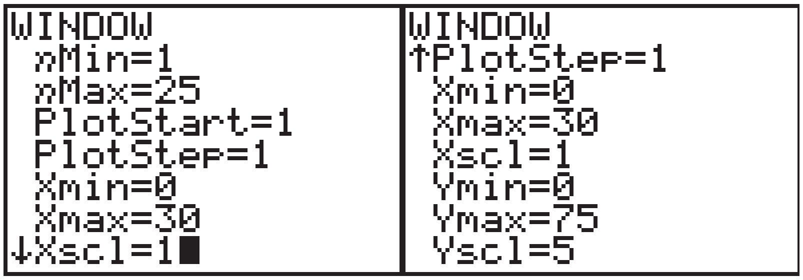
2. Use your graphing calculator to make a scatterplot of the number of seats in terms of the row number.

   There are several ways to generate the scatterplot on the graphing calculator.

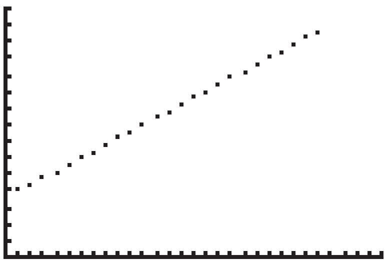
   One way is to use the Sequence Mode to enter the recursive formulas into the function editor.



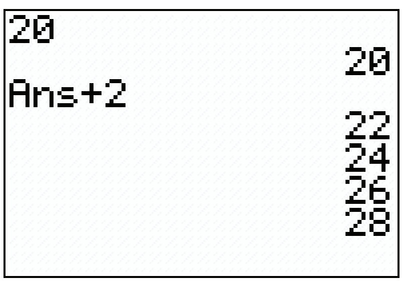
Set an appropriate viewing window, such as:



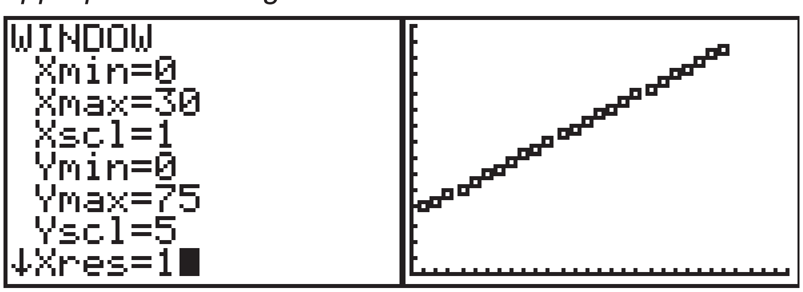
View the scatterplot.



Another way is to generate a table of values using home screen recursion that can be entered into the list editor in Function mode.



Enter the table of values into the List Editor, then generate the scatterplot using an appropriate viewing window.



3. Use your scatterplot, the recursive rule, or the problem situation to determine an explicit function rule for the number of seats for each row number.
*S = 2(*x *– 1) + 20 = 2x + 18*

4. Compare the constants in your recursive formula to the constants in your explicit function rule. How are they alike in terms of rates of change and initial starting conditions?

   The rate of change is 2 seats per row. This is addend in the recursive formula, and the multiplier in the explicit function rule.

   The initial condition is 20 seats in Row 1. For the recursive formula, this appears as the value for S_1. However, for the explicit function rule, this is the value (1, 20). For a linear function, the initial condition is actually the y-intercept, or the number of seats in Row 0, which does not really exist. Therefore, you need to subtract the rate of change from the first row, 20 – 2 = 18, so that the y-intercept value of the explicit function rule matches the situation.

5. How does the type of sequence generated by the recursive formula compare to the type of function generated by the explicit function rule? Include any domain restrictions in your description.

   The sequence is an arithmetic sequence, since there is a constant difference between the number of seats in each successive row. The function rule is a linear function, since there is a constant rate of change in the number of seats in each successive row.

   However, there really isn't a Row 1.5, so the domain of the linear function modeling this situation is restricted to natural numbers from 1 to 25.

Extension questions

- Suppose that the next section begins with 22 seats in Row 1, 24 seats in Row 2, 26 seats in Row 3, and so on. Write both a recursive formula and an explicit function rule that can be used to identify the number of seats, *S*, in row number *n*.
  $S_n = 2 + S_{n-1}$, $S_1 = 22$
  $S = 2(n - 1) + 22 = 2n + 20$

- Suppose that a section in Oiler Stadium, the home stadium of the Eagles' rival team, has 21 seats in Row 1, 24 seats in Row 2, 27 seats in Row 3, and so on. Write both a recursive formula and an explicit function rule that can be used to identify the number of seats, *S*, in row number *n*.
  $S_n = 3 + S_{n-1}$, $S_1 = 21$
  $S = 3(n - 1) + 21 = 3n + 18$

- In the auditorium at Eagle High School, the center orchestra section contains 30 rows of seats. The number of seats, *S*, in each row number, *n*, can be determined using the recursive formula, $S_n = 4 + S_{n-1}$, $S_1 = 20$. Write an explicit function rule to determine the number of seats, *S*, in row *n*.
  $S = 4(n - 1) + 20 = 4n + 16$